An Improved Viscous-Inviscid Interactive Method
and its Application to Ducted Propellers

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An Improved Viscous-Inviscid Interactive Method and
its Application to Ducted Propellers

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An Improved Viscous-Inviscid Interactive Method and its Application to Ducted Propellers

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Dedicated to my family
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An Improved Viscous-Inviscid Interactive Method and its Application to Ducted Propellers

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A two-dimensional viscous-inviscid interactive boundary layer method is applied to three dimensional problems of flow around ducts and ducted propellers. The idea is to predict the effects of fluid viscosity on three dimensional geometries, like ducts, using a two-dimensional boundary layer solver to avoid solving the fully three dimensional boundary layer equations, assuming that the flow is two-dimensional on individual sections of the geometry. The viscous-inviscid interactive method couples a perturbation potential based inviscid panel method with a two-dimensional viscous boundary layer solver using the wall transpiration model. The boundary layer solver used in the study solves for the integral boundary layer characteristics given the edge velocity distribution on the geometry. The viscous-inviscid coupling is applied in a stripwise manner but by including the interaction effects from other strips. An important development in this thesis is the consideration of effects of other strips in a more rational and accurate manner, leading to improved
results in the cases examined when compared to the results of a previous method. In particular, the effects of potentials due to other strips arising out of the three dimensional formulation are considered in this thesis. The validity of assuming two-dimensional flow along individual sections for application of viscous-inviscid coupling is investigated for the case of an open propeller by calculating the boundary layer characteristics in the direction normal to the assumed direction of two-dimensional flow from data obtained by RANS simulations. Also, a previous method which models the flow around the trailing edge of blunt hydrofoils has been improved and extended to three dimensional axisymmetric ducts. This method is applied to ducts with blunt and sharp trailing edges and to a ducted propeller. Correlations of results with experiments and simulations from RANS are shown.
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Nomenclature

Latin Symbols

\( C_D \) Dissipation coefficient \((1/\rho U^3) \int_0^\delta \tau \frac{\partial u}{\partial \eta} d\eta\)

\( C_f \) Skin-friction coefficient \( C_f = \tau_{wall}/(0.5 \rho U^2) \)

\( C_p \) Pressure coefficient, \( C_p = (P - P_0)/(0.5 \rho U^2) \)

\( C_T \) Shear stress coefficient \( C_T = \tau_{max}/(\rho U^2) \)

\( D \) Propeller diameter, \( D = 2R \)

\( f_{max}/C \) Maximum camber to chord ratio

\( G \) Green’s function

\( G_r \) Non-dimensional circulation, \( G_r = 100\Gamma/(2\pi RV_s) \)

\( H \) Shape factor, \( H = \delta^*/\theta \)

\( H^* \) Kinetic energy shape factor, \( \theta^*/\theta \)

\( H_K \) Kinematic shape parameter

\( J \) Advance ratio based on \( V_s \), \( J = V_s/(nD) \)

\( K_Q \) Torque coefficient, \( K_Q = Q/(\rho n^2 D^5) \)

\( K_T \) Thrust coefficient, \( K_T = T/(\rho n^2 D^4) \)

\( K_{TD} \) Thrust coefficient due to duct, \( K_{TD} = T_D/(\rho n^2 D^4) \)

\( \tilde{n} \) Transition disturbance amplification variable
$P$ \hspace{1em} Static pressure

$P_o$ \hspace{1em} Reference pressure at far upstream

$q$ \hspace{1em} Total velocity

$Q$ \hspace{1em} Torque

$R$ \hspace{1em} Propeller radius

$Re$ \hspace{1em} Reynolds number based on inflow velocity

$Re_{\theta}$ \hspace{1em} Reynolds number based on momentum thickness

$S_B$ \hspace{1em} Body surface

$S_W$ \hspace{1em} Wake surface

$t_{max}/C$ \hspace{1em} maximum thickness to chord ratio

$T$ \hspace{1em} Thrust

$T_D$ \hspace{1em} Thrust due to duct

$u$ \hspace{1em} Perturbation velocity

$U_e$ \hspace{1em} boundary layer edge velocity

$U_{in}$ \hspace{1em} local inflow velocity

$U_{inv}$ \hspace{1em} Total inviscid velocity

$U_{vis}$ \hspace{1em} Edge velocity

$u_{\tau}$ \hspace{1em} Wall shear velocity, $u_{\tau} = \sqrt{\tau_{wall}/\rho}$

$V_s$ \hspace{1em} Ship speed

$x, y, z$ \hspace{1em} ship fixed coordinates

$y^+$ \hspace{1em} non-dimensional wall distance, $y^+ = \frac{u_{\tau} y}{\nu}$
Greek Symbols

$\alpha$ angle of attack
$\Gamma$ Circulation on blade sections
$\delta^*$ displacement thickness, $\delta^* = \int (1 - \frac{u}{U_e})dz$
$\Delta t$ time step size
$\theta$ momentum thickness, $\theta = \int \frac{u}{U_e} (1 - \frac{u}{U_e})dz$
$\theta^*$ kinetic energy thickness, $\theta^* = \int \frac{u}{U_e} (1 - \frac{u^2}{U_e^2})dz$
$\rho$ fluid density
$\sigma$ Strength of boundary layer sources
$\tau$ Wall shear stress
$\nu$ kinematic viscosity of water
$\phi$ perturbation potential
$\phi^{inv}$ inviscid perturbation potential
$\phi^v$ viscous perturbation potential
$\Phi$ Total potential
$\omega$ propeller angular velocity
Uppercase Abbreviations

2-D Two dimensional
3-D Three dimensional
CPU Central processing unit
MIT Massachusetts Institute of Technology
NACA National Advisory Committee for Aeronautics
VLM Vortex-lattice method
RANS Reynolds-averaged Navier Stokes

Computer Program Names

PROPCAV A cavitating propeller potential flow solver based on BEM
XFOIL 2-D integral boundary layer analysis code
FLUENT A commercial RANS solver provided by ANSYS
ICEM CFD A commercial RANS meshing provided by ANSYS
MPUF-3A A VLM based code for solving flow around propellers
Chapter 1

Introduction

1.1 Background

Prediction of propeller performance is an important problem in the marine propeller industry. As the demand for bigger and more specialized vessels increase, more sophisticated propulsion systems are being installed that are more efficient. Boundary element methods are ideal to solve flow around propellers because they require much less computational resources and because of their ease of use. They are widely used in the design stage since they do not require discretizing the entire fluid domain. However these methods have a drawback because they essentially solve for the inviscid flow. The effects of fluid viscosity are neglected or are approximated by empirical corrections like a friction coefficient, which may or may not produce accurate results for all conditions. Including the effects of viscosity using RANS based calculations is computationally much more intensive and time-consuming. This makes RANS based methods unsuitable for hydrodynamic performance prediction in the design stage when a number of parameters may have to be changed. Hence it is desired to have a rational method that includes the effect of fluid viscosity in the calculations without resorting to viscous RANS based simulations. Interactive methods that couple the inviscid boundary element method
or any other inviscid solver with two-dimensional boundary layer solvers have been developed in the past few decades. Drela (1989) coupled an inviscid solver based on a vorticity based panel method with a two-dimensional integral boundary layer formulation for airfoils. The model by Drela uses a two equation lagged dissipation integral method and can handle separated flows. Milewski (1997) developed a 3-D fully simultaneous viscous/inviscid interactive scheme by coupling a potential based panel method and the 3D integral boundary layer equations, for 3D wetted hydrofoil and duct flows. However application of fully three dimensional boundary layer solver is complicated on complex geometries like propeller blades. The three dimensional boundary layer equations are usually written in orthogonal coordinate system on the surface and constructing an orthonormal grid on a convoluted surface like a propeller or a turbine blade is difficult. Solving boundary layer equations in three dimensions is also more time intensive and needs more computational resources. Jessup (1989) measured the boundary layer characteristics for Propeller DTMB 4119 and made an important observation that the boundary layer growth is mainly in the streamwise direction, i.e. the direction along constant radius on the blade. Following observations made by Jessup, Hufford (1992, 1994) coupled a three-dimensional panel method with a 2-D boundary layer solver in a strip wise manner., where the 2D boundary layer sources were replaced by 3D sources. The strips considered are the sections of the blade with constant radius. He assumed that the boundary layer growth in the spanwise direction of the blade can be neglected, so that the flow along each
individual strip is essentially two-dimensional. However the individual strips were completely decoupled when applying the boundary layer solver and hence effects of boundary layer sources on neighboring strips were not taken into consideration. Sun (2008) further simplified Hufford’s method and assumed that the boundary layer sources were two-dimensional sources. Sun (2008) also did not include the effect of other strips. Pan (2009), Pan & Kinnas (2011) also applied the viscous-inviscid method for open propellers and two-dimensional foils with blunt trailing edge by developing different schemes to model the flow downstream of the trailing edge.

Yu (2012) proposed a scheme to include the effects of boundary layer sources from other strips using three dimensional boundary layer sources by extending the two-dimensional viscous-inviscid coupling formulation to three dimensions. However the formulation was not completely accurate as it did not include important interaction effects from other strips.

1.2 Objective

The main objective of this thesis is to develop a more accurate 3D viscous-inviscid interactive method by coupling a 3D low-order perturbation potential based panel method code, PROPCAV, with a 2D integral boundary layer solver, XFOIL, to investigate the effects of fluid viscosity on the hydrodynamic performance of three dimensional geometries like ducts and ducted propellers.
1.3 Organization

This thesis is organized into six chapters.

Chapter 1 contains the literature review, the objective and the organization of this thesis.

Chapter 2 reviews a two-dimensional viscous-inviscid interactive method. The formulation and the coupling algorithm is discussed.

Chapter 3 presents the development of a three dimensional viscous-inviscid interactive method in a stripwise manner applicable to flow around ducts and propeller blades. The method follows the assumption of Hufford (1992, 1994), Sun (2008) and Yu (2012) in that the boundary layer along the span wise direction (direction on the surface normal to the streamwise direction) can be negligible. The method improves upon the formulation by Yu (2012) and considers the effects of neighboring strips in a more accurate manner. Then this method is validated through ducts with blunt and sharp trailing edges.

Chapter 4 discusses the validity of assumption of using two dimensional boundary layer solver for three dimensional flows. The three dimensional boundary layer equations are reviewed and the three dimensional terms are numerically evaluated using data from RANS simulation of an open propeller.

Chapter 5 improves the scheme proposed by Yu (2012) for hydrofoils with non-zero trailing edge and extends the method to axisymmetric ducts for application to ducted propellers having ducts with blunt trailing edges. A
scheme to apply viscous-inviscid interactive method on the ducted propeller to predict effects of viscosity on the duct is presented with preliminary results.

Chapter 6 presents the conclusions and recommendations for future work.
Chapter 2

Viscous-inviscid interactive method in two dimensions

In this chapter the viscous-inviscid interactive method for two dimensions is reviewed and the iterative coupling algorithm between the inviscid and viscous solutions is explained in detail. The two dimensional boundary layer equations in the integral form are presented. A perturbation potential based panel method is used for the inviscid solution in this formulation and it is coupled with the boundary layer solver XFOIL.

2.1 Inviscid formulation

Consider a two-dimensional hydrofoil subjected to uniform inflow $U_{in}$ at an angle of attack $\alpha$ as shown in Figure 2.1. Assuming inviscid and incompressible flow, the perturbation potential $\phi$ of the flow will satisfy the Laplace equation as in (2.1).

\[ \nabla^2 \phi = 0 \]  

(2.1)
The total velocity is the sum of the inflow and the perturbation velocity.

\[ q = \nabla \Phi \]
\[ = \nabla \Phi_{in} + \phi \quad \text{(2.2)} \]
\[ = U_{in} + \mathbf{u} \]

2.2 Boundary conditions

Since there cannot be any flow through the foil, by the Kinematic boundary condition,
\[ \frac{\partial \Phi}{\partial n} = 0 \]

\[ \frac{\partial \Phi_{in}}{\partial n} + \frac{\partial \phi}{\partial n} = 0 \]  
\[ \frac{\partial \Phi_{in}}{\partial n} = -\frac{\partial \phi}{\partial n} = -U_{in} \cdot n \]  

(2.3)

where \( n \) is the normal vector on the foil directed into the flow field. Also far away from the foil, there should be no effect of the foil, hence the perturbation velocity becomes zero.

\[ \nabla \phi = 0 \]  

(2.4)

To get a unique solution the Kutta condition is required that implies there are no sharp curvatures in the flow field. Thus the velocity should be finite at trailing edge.

\[ \nabla \phi = \text{finite at trailing edge} \]  

(2.5)

### 2.3 Integral equation for potential

Using Green’s theorem it can be shown that the perturbation potential will satisfy the following integral equation on the boundary,

\[ \frac{\partial \Phi}{\partial n} = 0 \]

\[ \frac{\partial \Phi_{in}}{\partial n} + \frac{\partial \phi}{\partial n} = 0 \]  
\[ \frac{\partial \Phi_{in}}{\partial n} = -\frac{\partial \phi}{\partial n} = -U_{in} \cdot n \]  

(2.3)
\[
\frac{\phi_M}{2} = \int_S \left[ -\phi_{M'} \frac{\partial G(M, M')}{\partial n_{M'}} + \frac{\partial \phi_{M'}}{\partial n_{M'}} G(M, M') \right] dS
\]
\[
- \int_{S_W} \Delta \phi_w \frac{\partial G(M, M')}{\partial n_{M'}} dS
\]
\[(2.6)\]

where \(M, M'\) are the control point and the variable points respectively and lie on the boundary.

\(G(M, M')\) is the Green’s function. \(G(M, M') = \frac{\ln(r)}{2\pi}\) in two dimensions and equals \(-\frac{1}{4\pi r}\) in three dimensions.

\(r\) is the separation between points \(M\) and \(M'\). \(S_B\) and \(S_W\) are the boundaries on the body and the wake respectively.

Thus the potential at a control point \(M\) can be considered to be the sum of potentials induced by dipoles of strength \(\phi_{M'}\) and sources of strength \(\frac{\partial \phi_{M'}}{\partial n_{M'}}\) on the foil and dipoles of strength \(\Delta \phi_w\) on the wake.

### 2.4 Discretized equations in inviscid flow

Consider the foil is discretized into \(N\) panels and the wake into \(N_W\) panels. Then integral equation (2.6) on the foil can be written as:

\[
\sum_{j=1}^{N} d_{ij} \phi_j = \sum_{j=1}^{N} s_{ij} \left( \frac{\partial \phi}{\partial n} \right)_j - w_i \Delta \phi_w
\]
\[(2.7)\]

where,

\[
\Delta \phi = \phi^N - \phi^1 + \mathbf{U}_{TE} \cdot \mathbf{r}_t = \Phi^N - \Phi^1 + \mathbf{U}_{TE} \cdot \mathbf{r}_t
\]
\[(2.8)\]

\(d_{ij}\) are the dipole induced influence coefficients by the \(j^{th}\) panel on the \(i^{th}\) control point.
$s_{ij}$ are the source induced influence coefficients by the $j^{th}$ panel on the $i^{th}$ control point.

$w_i$ are the dipole induced influence coefficients by wake on the $i^{th}$ control point.

$U_{TE}$ is the inflow velocity at the trailing edge of the foil and $r_t$ is the vector joining the control points of the lower and upper foil panel at the trailing edge. More details about this term can be found in Lee (1987). If the foil is adequately discretized and the angle of attack is small, the product $U_{TE} \cdot r_t$ can be neglected.

The dipole induced influence coefficients due to the wake can be combined with the dipole influence coefficients due to the foil and Equation (2.7) can be rewritten as Equation (2.9).

$$\sum_{j=1}^{N} a_{ij} \phi_j = \sum_{j=1}^{N} s_{ij} \left( \frac{\partial \phi}{\partial n} \right)_j$$

\(2.9\)

2.5 Viscous formulation

To simulate the effects of viscosity, a wall transpiration model is used. Extra sources of unknown strength $\sigma$ called as blowing sources are added on the foil and wake panels. The effect of these extra sources is to shift the potential flow away from the foil and to induce a normal velocity at the wall. The transpiration source strength is set such that the velocity normal to the wall in potential flow is equal to the normal velocity at the edge of the boundary layer in real flow. The source strengths can be shown to be related to the edge
velocity and displacement thickness as shown in Hufford (1992).

\[ \sigma = \frac{\partial m}{\partial s} = \frac{\partial (U_{vis} \delta^*)}{\partial s} \]  \hspace{2cm} (2.10)

where, \( m = U_{vis} \cdot \delta^* \) is the mass defect due to the boundary layer.

With the effect of the blowing sources, Equation (2.9) becomes,

\[ \sum_{j=1}^{N} a_{ij} \phi_j^v = \sum_{j=1}^{N} s_{ij} \left( \frac{\partial \phi}{\partial n} \right)_j + \sum_{j=1}^{N+N_W} b_{ij} \sigma_j \]  \hspace{2cm} (2.11)

where \( b_{ij} \) are the source induced influence coefficients by the \( j^{th} \) panel of the foil or wake on the \( i^{th} \) control point of the foil.

In matrix form,

\[ A \phi^v = S \frac{\partial \phi}{\partial n} + B \sigma \]  \hspace{2cm} (2.12)

where

\[ S = [s_{ij}] \]  \hspace{2cm} (2.13)

\[ B = [b_{ij}] \]  \hspace{2cm} (2.14)

\[ \frac{\partial \phi}{\partial n} = \left[ \left( \frac{\partial \phi}{\partial n} \right)_j \right] \]  \hspace{2cm} (2.15)
\[
\phi^v = [\phi^v_j] \tag{2.16}
\]

\[
\sigma = [\sigma_j] \tag{2.17}
\]

The potential obtained from Equation (2.22) are viscous potentials. Differentiating along the streamwise direction will give the viscous velocity or edge velocity.

The edge velocity on the foil can be related to the inviscid velocity by a correction involving the blowing source terms as shown in Hufford (1992) and Yu (2012).

\[
U_{i,vis} = U_{i,inv} + \sum_{j=1}^{N+N_w} c_{ij} \sigma_j \tag{2.18}
\]

or in matrix form,

\[
U_{i,vis} = U_{i,inv} + C\sigma \tag{2.19}
\]

where

\[
C = [c_{ij}] = \frac{\partial}{\partial s} (A^{-1}B) \tag{2.20}
\]

The viscous potential on the wake can be obtained by adding the influence of sources and dipoles on the foil and blowing sources on the foil and the wake.
$$\phi_{i,wake}^{v} = \sum_{j=1}^{N} a_{ij}^{w} \phi_{j}^{v} + \sum_{j=1}^{N} s_{ij}^{w} \left( \frac{\partial \phi}{\partial n} \right)_{j} + \sum_{j=1}^{N+N_{W}} b_{ij}^{w} \sigma_{j}$$ \hspace{1cm} (2.21)

$a_{ij}^{w}$ are the dipole induced influence coefficients by the $j^{th}$ panel of the body on the $i^{th}$ control point of the wake.

$s_{ij}^{w}$ are the source induced influence coefficients by the $j^{th}$ panel of the body on the $i^{th}$ control point of the wake.

$b_{ij}^{w}$ are the dipole induced influence coefficients by the $j^{th}$ panel of the body or wake on the $i^{th}$ control point of the wake.

In matrix form,

$$\phi_{i,wake}^{v} = A^{w} \phi^{v} + S^{w} \frac{\partial \phi}{\partial n} + B^{w} \sigma$$ \hspace{1cm} (2.22)

where,

$$B^{w} = \begin{bmatrix} b_{ij}^{w} \end{bmatrix}$$ \hspace{1cm} (2.23)

$$A^{w} = \begin{bmatrix} a_{ij}^{w} \end{bmatrix}$$ \hspace{1cm} (2.24)

$$S^{w} = \begin{bmatrix} s_{ij}^{w} \end{bmatrix}$$ \hspace{1cm} (2.25)

Thus, Equation (2.21) can be differentiated to get the viscous velocities on the wake and get a relation between inviscid and edge velocity on the wake as given in Yu (2012),
\[ U_{i,vis}^{wake} = U_{i,inv}^{wake} + \sum_{j=1}^{N+N_W} c_{ij}^{w} \sigma_j \]  

(2.26)

where

\[ [c_{ij}^w] = C^w = \frac{\partial}{\partial s} \left( A^w A^{-1} B + B^w \right) \]  

(2.27)

Hence, by combining Equations (2.18) and (2.26),

\[ U_{i,vis} = U_{i,inv} + \sum_{j=1}^{N+N_W} g_{ij} \sigma_j \]  

(2.28)

On foil,

\[ g_{ij} = c_{ij} \]  

(2.29)

On wake,

\[ g_{ij} = c_{ij}^w \]  

(2.30)

### 2.6 2-D Boundary layer equations

The two dimensional boundary layer equations solved in the boundary layer code XFOIL are given below. More details can be found in Drela (1989).

- **Momentum equation**

\[ \frac{\partial \theta}{\partial s} + (2 + H) \frac{\theta}{U_e} \frac{dU_e}{ds} = \frac{C_f}{2} \]  

(2.31)
• Kinetic energy equation

\[ \theta \frac{dH^*}{ds} + H^*(1 - H) \frac{\theta}{U_e} \frac{dU_e}{ds} = 2C_D - H^* \frac{C_f}{2} \quad (2.32) \]

• Closure

Closure for turbulent flows

\[ \frac{\delta}{C_\tau} \frac{dC_\tau}{ds} = 5.6 \left[ C_{\tau EQ}^{\frac{5}{3}} - C_{\tau}^{\frac{5}{3}} \right] \]

\[ + 2\delta \left\{ \frac{4}{3\delta^*} \left[ \frac{C_f}{2} - \left( \frac{H_k - 1}{6.7H_k} \right)^2 \right] - \frac{1}{U_e} \frac{dU_e}{ds} \right\} \quad (2.33) \]

Closure for laminar flows

\[ \frac{d\tilde{n}}{ds} = \frac{d\tilde{n}(H_k)}{dR_{e\theta}}(H_k, \theta) \frac{dR_{e\theta}}{ds} \quad (2.34) \]

2.7 Iterative scheme in two dimensions

The Equation (2.28) is an implicit equation in edge velocity. From the equation it can be seen that the edge velocity depends on the blowing sources, but the strength of blowing sources are themselves dependent on edge velocity and come from the boundary layer solution. Thus it has to be solved in an iterative manner. As an initial guess for edge velocity, the inviscid velocity is used in the boundary layer solver. The boundary layer solver uses the edge
velocity as a known distribution. Using this distribution the displacement thickness of boundary layer is calculated. These displacement thickness allow to evaluate the strengths of the blowing sources using Equation (2.10). These values of blowing sources are used in Equation (2.28) to update the value of edge velocity. Once the new edge velocity distribution is obtained, the boundary layer solver uses it as input and this process continues until convergence. This process is shown in a simple flowchart as shown in Figure 2.2.

Figure 2.2: Flowchart of the iterative viscous-inviscid interactive method in two dimensions.
Chapter 3

Viscous-inviscid interactive method in three dimensions

3.1 Background

In this chapter the Viscous-Inviscid interactive method is applied to three dimensional geometries by using the strip theory assumption. It is assumed that the growth of the boundary layer is only along one direction hence two-dimensional boundary layer equations are used. However the effects of other strips are taken into consideration keeping in mind the inherent three dimensional nature of the problem. Yu (2012) had included effects of neighboring strips but some interaction effects were missing. In this chapter the formulation for the interaction effects is developed in a rational way and it is shown that there are extra interaction effects present not only due to the blowing sources but also due to the viscous potentials on other strips. To validate the method, the formulation is simplified for an axisymmetric problem and the results predicted by the axisymmetric formulation are compared with those from RANS for axisymmetric ducts with blunt and sharp trailing edges. Also, the results obtained by applying the three dimensional interaction effects from the current scheme without the axisymmetric simplification are compared with those from the axisymmetric formulation for open duct cases.
3.2 Three dimensional formulation

The boundary layer growth is assumed to take place along the direction of a strip. For the case of propeller blades the direction of the strip is taken to be along a section of constant radius. For the case of a bare duct the flow would be axial, hence the streamwise direction would be the axial direction along the duct body or the meridional direction. It is assumed that the boundary layer growth in the other orthogonal direction would be small. The validity of this assumption will be further discussed in Chapter 4. The fully three dimensional boundary layer equations can be applied only for simple geometries like wings or ducts. However they are very time consuming because they have to solve equations in three dimensions. Hence when applicable, it is practical to use the strip theory assumption wherein only the two-dimensional boundary layer equations need to be solved.

Figure 3.1 shows a three dimensional body, divided into M strips in the spanwise direction. The boundary layer along the spanwise direction is neglected. The body is divided into N panels and its wake is divided into $N_w$ panels in the streamwise direction.

Similar to Equation (2.9) the equation for inviscid perturbation potential on the body can be written as,
The viscous effects can be considered by adding blowing sources on the body and the wake at all the panels. The equation for viscous perturbation potential on the duct can be written as,
\[
\begin{bmatrix}
A_{11} & A_{12} & \cdots & A_{1M} \\
A_{21} & A_{22} & \cdots & A_{2M} \\
\vdots & \vdots & \ddots & \vdots \\
A_{M1} & A_{M2} & \cdots & A_{MM}
\end{bmatrix}
\begin{bmatrix}
\phi_1^v \\
\phi_2^v \\
\vdots \\
\phi_M^v
\end{bmatrix}
= \\
\begin{bmatrix}
S_{11} & S_{12} & \cdots & S_{1M} \\
S_{21} & S_{22} & \cdots & S_{2M} \\
\vdots & \vdots & \ddots & \vdots \\
S_{M1} & S_{M2} & \cdots & S_{MM}
\end{bmatrix}
\begin{bmatrix}
\beta_1 \\
\beta_2 \\
\vdots \\
\beta_M
\end{bmatrix}
+ \\
\begin{bmatrix}
B_{11} & B_{12} & \cdots & B_{1M} \\
B_{21} & B_{22} & \cdots & B_{2M} \\
\vdots & \vdots & \ddots & \vdots \\
B_{M1} & B_{M2} & \cdots & B_{MM}
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\vdots \\
\sigma_M
\end{bmatrix}
\]

(3.2)

where

\(A_{PQ}, S_{PQ}\) are matrices of order \(NxN\) and denote the influence of dipoles and sources of \(Q^{th}\) strip on panels of the \(P^{th}\) strip respectively.

\(B_{PQ}\) is a matrix of order \(Nx(N+N_W)\) and denote the influence of blowing sources on duct and wake of \(Q^{th}\) strip on panels of the \(P^{th}\) strip.

\[
A_{PQ} = 
\begin{bmatrix}
a_{11}^{PQ} & a_{12}^{PQ} & \cdots & a_{1N}^{PQ} \\
a_{21}^{PQ} & a_{22}^{PQ} & \cdots & a_{2N}^{PQ} \\
\vdots & \vdots & \ddots & \vdots \\
a_{N1}^{PQ} & a_{N2}^{PQ} & \cdots & a_{NN}^{PQ}
\end{bmatrix}
\]

(3.3)

\[
S_{PQ} = 
\begin{bmatrix}
s_{11}^{PQ} & s_{12}^{PQ} & \cdots & s_{1N}^{PQ} \\
s_{21}^{PQ} & s_{22}^{PQ} & \cdots & s_{2N}^{PQ} \\
\vdots & \vdots & \ddots & \vdots \\
s_{N1}^{PQ} & s_{N2}^{PQ} & \cdots & s_{NN}^{PQ}
\end{bmatrix}
\]

(3.4)
\[ B_{PQ} = \begin{bmatrix} b_{11}^{PQ} & b_{12}^{PQ} & \cdots & b_{1,N+N_w}^{PQ} \\ b_{21}^{PQ} & b_{22}^{PQ} & \cdots & b_{2,N+N_w}^{PQ} \\ \vdots & \vdots & \ddots & \vdots \\ b_{N1}^{PQ} & b_{N2}^{PQ} & \cdots & b_{N,N+N_w}^{PQ} \end{bmatrix} \]  

(3.5)

\[ \beta_P = \left( \begin{array}{c} \left( \frac{\partial \phi}{\partial n} \right)_P^1 \\ \left( \frac{\partial \phi}{\partial n} \right)_P^2 \\ \vdots \\ \left( \frac{\partial \phi}{\partial n} \right)_P^N \end{array} \right) \]  

(3.6)

\[ \sigma_P = \begin{bmatrix} \sigma_P^1 \\ \sigma_P^2 \\ \vdots \\ \sigma_P^{N+N_w} \end{bmatrix} \]  

(3.7)

\[ \phi_P^v = \begin{bmatrix} \phi_1^{v,P} \\ \phi_2^{v,P} \\ \vdots \\ \phi_N^{v,P} \end{bmatrix} \]  

(3.8)

\[ \phi_P^{inv} = \begin{bmatrix} \phi_1^{inv,P} \\ \phi_2^{inv,P} \\ \vdots \\ \phi_N^{inv,P} \end{bmatrix} \]  

(3.9)

Substituting Equation (3.1) in Equation (3.2), we get,
Expanding along the $K^{th}$ row, we get the equation for viscous potentials on the $K^{th}$ strip as shown in Equation (3.11).

$$\sum_{j=1}^{M} A_{Kj} \phi_j^v = \sum_{j=1}^{M} A_{Kj} \phi_j^{inv} + \sum_{j=1}^{M} B_{Kj} \sigma_j$$

(3.11)

Hence,

$$A_{KK} \phi_K^v = A_{KK} \phi_K^{inv} + \sum_{J=1, J \neq K}^{M} [A_{KJ}] \left[ \phi_J^v - \phi_J^{inv} \right]$$

$$+ \sum_{J=1, J \neq K}^{M} B_{KJ} \sigma_J + B_{KK} \sigma_K$$

(3.12)

Solving for the viscous perturbation potential on the $K^{th}$ strip,

$$\phi_K^v = \phi_K^{inv} + \sum_{J=1, J \neq K}^{M} \left[ A_{KK}^{-1} \right] [A_{KJ}] \left[ \phi_J^v - \phi_J^{inv} \right]$$

$$+ A_{KK}^{-1} B_{KK} \sigma_K + \sum_{J=1, J \neq K}^{M} A_{KK}^{-1} B_{KJ} \sigma_J$$

(3.13)
The total potential is the sum of inflow and the perturbation potential,

\[
\phi_v^K + \Phi_{K, inflow} = \phi_v^{invK} + \Phi_{K, inflow} \\
+ \sum_{J=1, J\neq K}^{M} [A_{KK}^{-1}] [A_{KJ}] [\phi_v^J - \phi_v^{invJ}] \\
+ \sum_{J=1, J\neq K}^{M} A_{KK}^{-1} B_{KJ} \sigma_J \tag{3.14}
\]

\[
\Phi_v^K = \Phi_v^{invK} + \sum_{J=1, J\neq K}^{M} [A_{KK}^{-1}] [A_{KJ}] [\phi_v^J - \phi_v^{invJ}] \\
+ A_{KK}^{-1} B_{KK} \sigma_K + \sum_{J=1, J\neq K}^{M} A_{KK}^{-1} B_{KJ} \sigma_J \tag{3.15}
\]

Differentiating along the streamwise direction of the \(K^{th}\) strip gives the edge velocity on that strip,

\[
U_v^K = U_v^{invK} + D_K A_{KK}^{-1} B_{KK} \sigma_K \\
+ \sum_{J=1, J\neq K}^{M} D_K [A_{KK}^{-1}] [A_{KJ}] [\phi_v^J - \phi_v^{invJ}] \\
+ \sum_{J=1, J\neq K}^{M} [D_K] A_{KK}^{-1} B_{KJ} \sigma_J \tag{3.16}
\]

Hence,

\[
U_v^K = U_v^{invK} + Q_{KK} \sigma_K \\
+ \sum_{J=1, J\neq K}^{M} [H_{KJ}] [\phi_v^J - \phi_v^{invJ}] + \sum_{J=1, J\neq K}^{M} Q_{KJ} \sigma_J \tag{3.17}
\]
where, $Q_{KJ} = D_K A_{KK}^{-1} B_{KJ}$ and $H_{KJ} = D_K A_{KK}^{-1} A_{KJ}$.

$D_J$ is the differentiation matrix on the $J^{th}$ strip.

Using the relation between the blowing source strength and the mass defects, Equation (2.10),

$$U^v_K = \left( U^{inv}_K + Q_{KK} D_K m_K \right) + \sum_{J=1, J \neq K}^{M} \left[ H_{KJ} \left( \phi^u_J - \phi^{inv}_J \right) \right] + \sum_{J=1, J \neq K}^{M} Q_{KJ} D_J m_J \quad (3.18)$$

3.2.1 Edge velocity expression on the body

Thus the expression for edge velocity on the $K^{th}$ strip can be written in the following form,

$$U^v_K = U^{inv}_K + \sum_{J=1, J \neq K}^{M} \left[ H_{KJ} \left( \phi^u_J - \phi^{inv}_J \right) \right] + T_{KK} m_K + \sum_{J=1, J \neq K}^{M} T_{KJ} m_J \quad (3.19)$$

where, $T_{KJ} = Q_{KJ} D_J$ and $m_J = \left( U^v \cdot \delta^* \right)_J$.

Note: In Yu (2012), the three dimensional formulation was extrapolated from the two-dimensional formulation by adding extra terms from other strips. In
his formulation, the interaction term \( \sum_{J=1, J \neq K}^{M} [H_{KJ}] [\phi_{J}^{v} - \phi_{J}^{in}] \) due to viscous potentials on other strips were not considered in the above equation for edge velocity on the body. Hence this formulation is more accurate to properly account for the three dimensional effects.

### 3.2.2 Edge velocity expression on the wake

The viscous potentials on the wake can be considered to be the sum of potentials induced by the three dimensional sources and dipoles on the duct and blowing sources on the wake control points. The expression for perturbation potential on the wake can be written as,

\[
\phi_{v,wake}^{w} = \sum_{J=1}^{M} A_{KJ}^{w} \phi_{J}^{v} + \sum_{J=1}^{M} S_{KJ}^{w} \left( \frac{\partial \phi}{\partial n} \right)_{J}^{v} + \sum_{J=1}^{M} B_{KJ}^{w} \sigma J
\] (3.20)

The total potential includes the inflow potential,

\[
\Phi_{K,wake}^{v} = \Phi_{K,wake, in}^{v} + \sum_{J=1}^{M} A_{KJ}^{w} \phi_{J}^{v}
\]

\[
+ \sum_{J=1}^{M} S_{KJ}^{w} \left( \frac{\partial \phi}{\partial n} \right)_{J}^{v} + \sum_{J=1}^{M} B_{KJ}^{w} \sigma J
\] (3.21)

The edge velocities on the wake can be obtained by differentiating in the streamwise direction along the wake.
\[
U_{K}^{v,wake} = U_{K}^{wake,in} + \sum_{J=1}^{M} D_{J} A_{KJ}^{w} \phi_{J}^{v} + \sum_{J=1}^{M} D_{J} S_{KJ}^{w} \left( \frac{\partial \phi}{\partial n} \right)_{J} + \sum_{J=1}^{M} D_{J} B_{KJ}^{w} \sigma_{J} 
\]

where, \( D_{J} \) is the differentiation matrix on the \( J^{th} \) strip.

Using the relation between the blowing source strength and the mass defects, Equation (2.10),

\[
U_{K}^{v,wake} = U_{K}^{wake,in} + \sum_{J=1}^{M} D_{J} A_{KJ}^{w} \phi_{J}^{v} + \sum_{J=1}^{M} D_{J} S_{KJ}^{w} \left( \frac{\partial \phi}{\partial n} \right)_{J} + \sum_{J=1}^{M} D_{J} B_{KJ}^{w} \sigma_{J} \]

(3.23)

Using the expression for viscous potentials on the body from Equation (3.13),

\[
U_{K}^{v,wake} = \sum_{J=1}^{M} D_{J} S_{KJ}^{w} \left( \frac{\partial \phi}{\partial n} \right)_{J} + \sum_{J=1}^{M} D_{J} A_{KJ}^{w} \phi_{J}^{v} \]

- \( A_{1P}^{-1} B_{JJP} \phi_{JP}^{v} \)

(3.24)

\[
+ \sum_{J=1}^{M} D_{J} B_{KJ}^{w} \sigma_{J} \]

+ \( D_{K} B_{KK}^{w} D_{K} m_{K} \)

\[
+ \sum_{J=1}^{M} D_{J} B_{KJ}^{w} D_{J} m_{J} \]

+ \( D_{K} B_{KK}^{w} D_{K} m_{K} \)
In Yu (2012), effectively the interaction term \( \sum_{P=1,P\neq I}^M [A_{JJ}^{-1}] [A_{JP}] [\phi_P^v - \phi_P^{inv}] \) due to viscous potentials on other strips were not considered in the above equation for wake edge velocities. Hence this formulation is more accurate to properly account for the three dimensional effects.

### 3.3 Iterative scheme in three dimensions

Since the viscous potentials on other strips and the blowing source strengths on other strips are not known, the viscous-inviscid interactive method has to be solved iteratively.

For the initial guess the inviscid velocity is used as edge velocity on all the strips and the viscous-inviscid coupling is solved on all the strips without considering effects of other strips. This gives an initial guess for the blowing sources. Also the viscous potentials on all the strips are updated using the values of blowing sources from the boundary layer solver. Then the edge velocity on each strip is updated using Equation (3.19) on the duct and Equation (3.23) on the wake wherein the interaction effects due to viscous potentials and blowing sources are taken into account. With the updated edge velocities the above process is continued until convergence. The process in explained in a flowchart in Figure 3.2.
For initial guess use $U_{\text{edge}} = \text{inviscid velocity}$

Calculate $\delta^*$ from edge velocity distribution using on each strip using 2-D boundary layer solver

Calculate blowing source strengths $\sigma$ on the strip

Update edge velocities using new strengths $\sigma$ on the strip

Solution on strip converged?

Yes

Update viscous potential on body and wake on all the strips

Update edge velocities on all strips by including all interaction effects

Edge velocity on all strips converged?

Yes

Save edge velocities and exit

No

Figure 3.2: Flowchart of viscous-inviscid interactive method in 3-D including interaction effects of blowing sources and potential difference from other strips.
3.4 Formulation for axisymmetric flows

If the body geometry and the flow is axisymmetric then the above three dimensional formulation can be simplified. Since the flow is axisymmetric the solutions at all the strips will be the same, hence only one strip needs to be solved for. The two-dimensional influence coefficients of one panel at a control point can be replaced by the three-dimensional influence coefficients of an annular panel ring on the said control point in Equation (2.9). This is useful for validating the general three dimensional formulation developed in the previous section.

The results from the axisymmetric formulation are compared for two cases. A duct with blunt trailing edge and a long duct with a sharp trailing edge are used to test the formulation.

3.5 Results for duct with sharp trailing edge

The duct geometry used is shown in the Figure 3.3. The duct is placed in a uniform inflow at zero angle of attack. A long duct was selected because the effects of viscosity were found to be more pronounced for a long duct. The transition location is kept at 10 % from the leading edge on both the pressure and suction side of the duct. The Reynolds number based on chord length is $4e6$. The discretization of the duct and the wake for applying the viscous-inviscid interactive method is shown in Figure 3.4.
Figure 3.3: Geometry of the 'long' duct. Note the chord length is much greater than the radius.

Figure 3.4: Paneling on the duct and the wake in the viscous-inviscid solver. The figure shows 90 streamwise (chordwise) panels and 50 spanwise (circumferential) panels on the duct and 50 chordwise panels on the wake.
To validate the results from the three dimensional formulation, an axisymmetric RANS simulation was done using Fluent. The mesh was made using ICEM CFD meshing software. The details of the RANS grid are given in Table 3.1. The domain and boundary conditions of the RANS mesh are shown in Figure 3.5.

Table 3.1: Information of the RANS (ANSYS FLUENT) case for duct with sharp trailing edge.

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<td>Pressure correction scheme</td>
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<td>Spatial discretization</td>
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</table>

Figure 3.5: Domain and boundary conditions of the RANS (ANSYS FLUENT) mesh for the axisymmetric long duct case.
To capture the boundary layer on the duct, the mesh is made very fine near the duct. Enhanced wall functions have been used near the duct as in the previous case and the non-dimensional wall distance is controlled in the $O(1)$. The details of the mesh are shown in Figures 3.6 to 3.8 and the non-dimensional wall distance on the duct is shown in Figure 3.9.

![Mesh created for the RANS simulation for the duct with sharp trailing edge.](image)

Figure 3.6: Mesh created for the RANS simulation for the duct with sharp trailing edge.

The pressure distribution on the duct obtained from RANS and from the axisymmetric viscous-inviscid interactive method are compared in Figure
Figure 3.7: Grid details near the leading edge of the long duct.
Figure 3.8: Grid details near the trailing edge of the long duct.
Figure 3.9: Non dimensional wall distance $y^+$ on the duct with sharp trailing edge.
3.10. It can be seen that the correlation of pressures from the viscous-inviscid interactive method and RANS is very good, indicating that the method can correctly predict the effects of viscosity.

Figure 3.10: Comparison of pressure coefficient on the duct with sharp trailing edge obtained from viscous-inviscid interactive scheme and RANS.

To compare the boundary layer characteristics, normals were created on the duct at various locations and the velocity profiles on those normals were extracted. The normals created on the duct are shown in Figure 3.11. On the points on these normals, the velocity components in the streamwise direction
(along the duct contour) as shown in Figures 3.12 and 3.13 were calculated. Using these velocity profiles at various points on the duct, the displacement and momentum thicknesses were calculated.

Figure 3.11: Normal vectors created on the duct with sharp trailing edge. Created to obtain velocity information from the RANS solution.
Figure 3.12: Velocity profiles on the inner side of the duct with sharp trailing edge.
Figure 3.13: Velocity profiles on the outer side of the duct with sharp trailing edge.
The boundary layer characteristics obtained from axisymmetric viscous-inviscid formulation are compared with those calculated from RANS in Figures 3.14 to 3.18. The agreement is good considering the approximations involved in the integral boundary layer formulation. Also it should be noted that RANS results may not be completely accurate because of the errors arising from the need to model the turbulent stress terms. However, RANS results can be used as a guide for comparison of results from the current scheme. From the figures below it can be seen that the axisymmetric formulation can predict boundary layer characteristics with reasonable accuracy.

To further validate the results a fully three dimensional Fluent mesh was created with the particulars given in Table 3.2. The results from the axisymmetric and fully three dimensional run were found to be the same.

Table 3.2: Information of the RANS (ANSYS FLUENT) fully 3-D case for duct with sharp trailing edge.

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Figure 3.14: Comparison of displacement thickness $\delta^*$ on the inner side of the duct with sharp trailing edge obtained from viscous inviscid interactive method and RANS.
Figure 3.15: Comparison of displacement thickness \( \delta^* \) on the outer side of the duct with sharp trailing edge obtained from viscous-inviscid interactive method and RANS.
Figure 3.16: Comparison of momentum thickness $\theta$ on the inner side of the duct with sharp trailing edge obtained from viscous-inviscid interactive method and RANS.
Figure 3.17: Comparison of momentum thickness $\theta$ on the outer side of the duct with sharp trailing edge obtained from viscous-inviscid interactive method and RANS.
Figure 3.18: Comparison of skin friction coefficient $C_f$ on the duct with sharp trailing edge obtained from viscous-inviscid interactive method and RANS.
3.5.1 Convergence studies

To check that the results from the viscous-inviscid formulation are grid independent, systematic convergence tests were performed. In the first study, the number of chordwise panels on the duct while keeping the number of spanwise panels a constant. The results of this study are shown in Figure 3.19.

In the second study, the number of chordwise panels on the duct were kept constant and the number of spanwise panels were changed. The results of this study are shown in Figure 3.20.

It can be seen from the results from the convergence study for the long duct case that the viscous-inviscid iterative scheme developed is robust and reliable.
Figure 3.19: Convergence study of the viscous-inviscid interactive scheme by changing the number of chordwise elements on the duct with sharp trailing edge.
Figure 3.20: Convergence study of the viscous-inviscid interactive scheme by changing the number of spanwise elements on the duct with sharp trailing edge.
3.6 Results for duct with blunt trailing edge

The three dimensional viscous-inviscid interactive method is applied on a axisymmetric duct with a blunt trailing edge. The duct studied in this case has a sharp curvature at the inner side near its trailing edge which leads to separated flow on the inner side of the duct. The potential solver will not be able to solve the flow behind blunt trailing edge due to presence of large recirculation region created by the flow separation. Hence the trailing edge of the duct needs to be modified using an extension scheme. The original shape and the modified shape of the duct section are shown in Figure 3.21. The details of the extension scheme used to modify the trailing edge of the duct are provided in Chapter 5.

![Figure 3.21: Sections of the duct with original and modified geometry. The geometry is modified to allow the application of BEM. Original geometry is in solid line and the modified geometry is shown by the dashed line.](image)
The three dimensional geometry of the duct is shown in Figure 3.22. The duct is placed in uniform inflow at zero degree angle of attack. The transition location is forced at 10 % on both the pressure and suction side of the duct. The Reynolds number is 1,000,000.

Figure 3.22: Three dimensional geometry of the duct after extending the trailing edge using extension scheme.
To validate the results from the three dimensional axisymmetric formulation, an axisymmetric RANS simulation was done using ANSYS FLUENT software. The mesh was made using ANSYS ICEM CFD meshing software. Since RANS is able to handle separated flow regions, unlike the inviscid solver, there is no need to extend the trailing edge of the blunt duct in the RANS mesh. Hence the RANS grid is made by using the original duct geometry so that the results from the viscous-inviscid formulation can be compared with results that we physically expect to observe. The domain of mesh and boundary conditions are shown in Figure 3.23. The details of the RANS grid are given in Table 3.3.

![Figure 3.23: Domain and boundary conditions for the RANS (ANSYS FLUENT) mesh for the axisymmetric duct case with blunt trailing edge.](image-url)
Table 3.3: Information of the axisymmetric RANS (ANSYS FLUENT) case for duct with blunt trailing edge.

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To capture the boundary layer on the duct the mesh is made very fine near the duct. The grid details near the duct are shown in Figures 3.24 to 3.26. Enhanced wall functions have been used near the duct and the non-dimensional wall distance is controlled in the recommended range of 1 to 5. The non-dimensional wall distance on the duct from the RANS simulation is shown in the Figure 3.27.
Figure 3.24: RANS grid details near the duct for the axisymmetric duct case with blunt trailing edge.
Figure 3.25: RANS grid details near the leading edge of the duct for the axisymmetric duct case with blunt trailing edge.
Figure 3.26: RANS grid details near the trailing edge of the duct for the axisymmetric duct case with blunt trailing edge.
Figure 3.27: Non dimensional wall distance $y^+$ on the duct with blunt trailing edge.
The pressures on the duct calculated by using the axisymmetric viscous-inviscid iterative scheme are compared with the results from the RANS simulations and the comparison is shown in Figure 3.28. In the same figure, the inviscid pressure distribution on the duct as calculated by the inviscid panel method is also shown.

Figure 3.28: Comparison of pressure coefficient on the duct with blunt trailing edge obtained from the axisymmetric viscous-inviscid interactive scheme and RANS.
It can be seen that there is a significant difference between the viscous and inviscid pressures and we can conclude that the viscous-inviscid iterative scheme developed here can predict this effect with very good accuracy.

To compare the boundary layer characteristics, normals were created on the duct at various locations both on the inner and outer side of the duct, and the velocity profiles from RANS solution on points along those normals were extracted. The normals created on the duct are shown in the Figure 3.29. The velocity components in the streamwise direction as shown in Figure 3.30 and Figure 3.31 were calculated at points on these normals and the displacement and momentum thicknesses at different locations on the duct were calculated from these velocity profiles.

![Figure 3.29: Normal vectors created on the duct with blunt trailing edge to obtain velocity information from RANS solution.](image)
Figure 3.30: Velocity profiles on the inner side of the duct with blunt trailing edge from axisymmetric RANS solution.
Figure 3.31: Velocity profiles on the outer side of the duct with blunt trailing edge from axisymmetric RANS solution.
The boundary layer characteristics obtained from the axisymmetric
viscous-inviscid formulation are compared with those calculated from RANS
data in Figures 3.32 to 3.36. The agreement is good considering the approxi-
mations involved in the integral boundary layer formulation. From the figures
below it can be seen that the axisymmetric formulation can predict boundary
layer characteristics with reasonable accuracy.

It is to be noted that we are interested only in the results over the actual
length of the duct. The results obtained on the extension beyond the location
of the actual trailing edge are not of interest physically. In the figures below,
the results have been shown until the actual trailing edge location.
Figure 3.32: Comparison of displacement thickness $\delta^*$ on the inner side of the duct with blunt trailing edge, obtained from viscous inviscid interactive method and RANS.
Figure 3.33: Comparison of displacement thickness $\delta^*$ on the outer side of the duct with blunt trailing edge, obtained from viscous-inviscid interactive method and RANS.
Figure 3.34: Comparison of momentum thickness $\theta$ on the inner side of the duct with blunt trailing edge, obtained from viscous-inviscid interactive method and RANS.
Figure 3.35: Comparison of momentum thickness $\theta$ on the outer side of the duct with blunt trailing edge, obtained from viscous-inviscid interactive method and RANS.
Figure 3.36: Comparison of skin friction coefficient $C_f$ on the duct with blunt trailing edge, obtained from viscous-inviscid interactive method and RANS.
3.6.1 Convergence studies

To check that the results from the viscous-inviscid formulation are grid independent, systematic convergence tests were performed. In the first study, the number of chordwise panels on the duct while keeping the number of spanwise panels a constant. The results of this study are shown in Figure 3.37.

Figure 3.37: Convergence study of the viscous-inviscid interactive scheme by changing the number of chordwise elements on the duct with blunt trailing edge.
In the second study, the number of chordwise panels on the duct were kept constant and the number of spanwise panels were changed. The results of this study are shown in Figure 3.38.

Figure 3.38: Convergence study of the viscous-inviscid interactive scheme by changing the number of spanwise elements on the duct with blunt trailing edge.
3.7 Results using the general three dimensional formulation

The results from the three dimensional formulation are compared with those from axisymmetric formulation for an open duct case with a sharp trailing edge as shown in Section 3.5. In the Figure 3.39, the pressures on the duct from the axisymmetric and general three dimensional formulation are compared.

It can be seen, that the results from the general three dimensional formulation are very close to those from axisymmetric formulation as expected and validates the general formulation.
Figure 3.39: Comparison of pressures on duct with sharp trailing edge from axisymmetric viscous-inviscid formulation and general three dimensional viscous-inviscid formulation.
3.7.1 Convergence studies

Convergence studies are presented for the general three dimensional formulation by changing the number of chordwise and spanwise elements as shown in Figures 3.40 and 3.41.

Figure 3.40: Convergence study of the fully three dimensional viscous-inviscid interactive scheme by changing the number of chordwise elements on the duct with sharp trailing edge.
Figure 3.41: Convergence study of the fully three dimensional viscous-inviscid interactive scheme by changing the number of spanwise elements on the duct with sharp trailing edge.
Chapter 4

Boundary layer characteristics in the spanwise direction

4.1 Motivation

When we use the viscous-inviscid interactive method in a stripwise manner, we assume that the spanwise boundary layer is negligible. To verify the validity of this assumption, in this chapter the boundary layer characteristics in spanwise direction are evaluated numerically from RANS simulation data of an open propeller. The boundary layer characteristics in the streamwise direction (along curves of constant radii) and spanwise (tangential) direction are compared and analyzed for two different rotational speeds of propeller NSRDC4381 at different blade radii. Also correlations between the boundary layer characteristics obtained from using a previous viscous-inviscid interactive method and RANS results are presented.

4.2 Three dimensional boundary layer equations

Consider $x_1$ and $x_2$ are orthogonal direction on a surface and $x_3$ is the wall normal direction perpendicular to the surface, then the boundary layer equations for three dimensional flows can be written as given in White (1974).
Along the \( x_1 \) direction,

\[
\frac{1}{V^2 h_1} \frac{\partial}{\partial x_1} (V^2 \theta_{11}) + \frac{\delta^*_1}{V h_1} \frac{\partial U_1}{\partial x_1} + \frac{1}{V^2 h_2} \frac{\partial}{\partial x_2} (V^2 \theta_{12}) + \frac{\delta^*_2}{V h_2} \frac{\partial U_1}{\partial x_2} \\
- K_2 \left( \theta_{12} + \theta_{21} + \frac{U_2 \delta^*_1}{V} \right) + K_1 \left( \theta_{22} - \theta_{11} + \frac{U_2 \delta^*_2}{V} \right) = \frac{\tau_{w_1}}{\rho V^2}
\]

(4.1)

Along the \( x_2 \) direction,

\[
\frac{1}{V^2 h_1} \frac{\partial}{\partial x_1} (V^2 \theta_{21}) + \frac{\delta^*_1}{V h_1} \frac{\partial U_1}{\partial x_1} + \frac{1}{V^2 h_2} \frac{\partial}{\partial x_2} (V^2 \theta_{22}) + \frac{\delta^*_2}{V h_2} \frac{\partial U_2}{\partial x_2} \\
- K_1 \left( \theta_{12} + \theta_{21} + \frac{U_1 \delta^*_2}{V} \right) + K_2 \left( \theta_{11} - \theta_{22} + \frac{U_1 \delta^*_1}{V} \right) = \frac{\tau_{w_2}}{\rho V^2}
\]

(4.2)

where, \( \delta^*_1 \) is the displacement thickness along the \( x_1 \) direction, \( \delta^*_2 \) is the displacement thickness along the \( x_2 \) direction.

\[
\delta^*_1 = \frac{1}{V} \int_0^\delta (U_1 - u_1) \, dx_3 \quad (4.3)
\]

\[
\delta^*_2 = \frac{1}{V} \int_0^\delta (U_2 - u_2) \, dx_3 \quad (4.4)
\]

\( \theta_{11} \) is the momentum thickness along \( x_1 \) direction, \( \theta_{22} \) is the momentum thickness along \( x_2 \) direction.

\[
\theta_{11} = \frac{1}{V^2} \int_0^\delta u_1 (U_1 - u_1) \, dx_3 \quad (4.5)
\]
\[ \theta_{22} = \frac{1}{V^2} \int_{0}^{\delta} u_2 (U_2 - u_2) \, dx_3 \]  

(4.6)

\[ \theta_{12} = \frac{1}{V^2} \int_{0}^{\delta} u_2 (U_1 - u_1) \, dx_3 \]  

(4.7)

\[ \theta_{21} = \frac{1}{V^2} \int_{0}^{\delta} u_1 (U_2 - u_2) \, dx_3 \]  

(4.8)

\( K_1 \) and \( K_2 \) are the geodesic curvatures of the lines \( x_1 = \text{constant} \) and \( x_2 = \text{constant} \) respectively.

\[ K_1 = - \frac{1}{h_1 h_2} \frac{\partial h_2}{\partial x_1} \]  

(4.9)

\[ K_2 = - \frac{1}{h_1 h_2} \frac{\partial h_1}{\partial x_2} \]  

(4.10)

### 4.3 Calculation method using RANS for open propeller

For calculating the boundary layer data on the propeller blades, an existing RANS solution data of propeller NSRDC4381 by Tian and Kinnas (2011) is used. It is a five bladed propeller. The domain of the RANS (ANSYS FLUENT) simulation and the mesh near the blade is shown in Figures 4.1 and 4.2.
Figure 4.1: Domain and boundary conditions of the RANS mesh for propeller NSRDC4381. Taken from Sharma (2011).

Figure 4.2: RANS mesh on the blade of propeller NSRDC4381.
To get the streamwise direction, two slices of radii $r/R=0.6$ and $r/R=0.95$ are made on the blade. The radius $r/R=0.95$ is selected to gather information near the blade tip. The boundary layer characteristics are calculated at these two radii. As mentioned previously in the viscid-iviscid formulation we assume growth of boundary layer in streamwise direction. Here the direction along the curves at $r/R=0.6$ and $r/R=0.95$ are the streamwise direction.

The details of the RANS mesh are given in Table 4.1.

Table 4.1: Information of the RANS (ANSYS FLUENT) simulation for open propeller NSRDC4381.

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Three points are selected each on pressure and suction side of the blade at both radii to calculate the boundary layer variables at those points. The locally orthonormal direction to the streamwise direction at those points are calculated and the normal vectors to the blade surface at those points are calculated. The Figures 4.3 and 4.4 show the normals drawn to the blade surface at two different radii.
Figure 4.3: Normal directions at $r/R = 0.6$ on the blade of propeller NSRDC4381.

Figure 4.4: Normal directions at $r/R = 0.95$ on the blade of propeller NSRDC4381.
The blade sections at the two radii are shown in Figures 4.5 and 4.6.

Figure 4.5: Blade section at $r/R = 0.6$ on the blade of propeller NSRDC4381.

Figure 4.6: Blade section at $r/R = 0.95$ on the blade of propeller NSRDC4381.
4.4 Velocity profiles at different sections for open propeller

The boundary layer characteristics are calculated from two different advance ratios, J=0.5 and J=0.889. The velocity profiles in the locally streamwise and spanwise direction at the normals of the selected points are shown below. It is found as expected that the spanwise component of velocity is much lesser than the streamwise component.

Figure 4.7: Streamwise and tangential velocity profiles at point 1 at r/R= 0.6 of propeller NSRDC4381 for J=0.889
Figure 4.8: Streamwise and tangential velocity profiles at point 2 at r/R = 0.6 of propeller NSRDC4381 for J = 0.889
Figure 4.9: Streamwise and tangential velocity profiles at point 3 at $r/R = 0.6$ of propeller NSRDC4381 for $J = 0.889$
Figure 4.10: Streamwise and tangential velocity profiles at point 4 at r/R=0.6 of propeller NSRDC4381 for J=0.889
Figure 4.11: Streamwise and tangential velocity profiles at point 5 at r/R= 0.6 of propeller NSRDC4381 for J=0.889
Figure 4.12: Streamwise and tangential velocity profiles at point 6 at $r/R=0.6$ of propeller NSRDC4381 for $J=0.889$
Figure 4.13: Streamwise and tangential velocity profiles at point 1 at r/R=0.95 of propeller NSRDC4381 for J=0.889
Figure 4.14: Streamwise and tangential velocity profiles at point 2 at r/R=0.95 of propeller NSRDC4381 for J=0.889
Figure 4.15: Streamwise and tangential velocity profiles at point 3 at $r/R=0.95$ of propeller NSRDC4381 for $J=0.889$
Figure 4.16: Streamwise and tangential velocity profiles at point 4 at $r/R=0.95$ of propeller NSRDC4381 for $J=0.889$
Figure 4.17: Streamwise and tangential velocity profiles at point 5 at r/R=0.95 of propeller NSRDC4381 for J=0.889
Figure 4.18: Streamwise and tangential velocity profiles at point 6 at $r/R=0.95$ of propeller NSRDC4381 for $J=0.889$
Figure 4.19: Streamwise and tangential velocity profiles at point 1 at $r/R=0.6$ of propeller NSRDC4381 for $J=0.5$
Figure 4.20: Streamwise and tangential velocity profiles at point 2 at \( r/R = 0.6 \) of propeller NSRDC4381 for \( J=0.5 \)
Figure 4.21: Streamwise and tangential velocity profiles at point 3 at $r/R=0.6$ of propeller NSRDC4381 for $J=0.5$
Figure 4.22: Streamwise and tangential velocity profiles at point 4 at r/R=0.6 of propeller NSRDC4381 for J=0.5
Figure 4.23: Streamwise and tangential velocity profiles at point 5 at \( r/R = 0.6 \) of propeller NSRDC4381 for \( J=0.5 \)
Figure 4.24: Streamwise and tangential velocity profiles at point 6 at \( r/R = 0.6 \) of propeller NSRDC4381 for \( J = 0.5 \).
4.5 Results of boundary layer characteristics in the spanwise direction

The results from calculation of the boundary layer variables in equations (4.3) to (4.8) are presented in Figures 4.25 to 4.28. It can be seen that for most of the cases the spanwise boundary layer component is an order of magnitude or lesser than the streamwise component. There are some exceptions near the tip. In general from the RANS results it can be said that applying boundary layer on the propeller blade in a stripwise manner would be a reasonable assumption considering the computational and modeling difficulties of fully three dimensional boundary layer equations.

| Point | $|\delta_1^* / R|$ | $|\delta_2^* / R|$ | $|\theta_{11} / R|$ | $|\theta_{22} / R|$ | $|\theta_{12} / R|$ | $|\theta_{21} / R|$ |
|-------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| 1     | 1.16E-03           | 3.41E-05           | 6.42E-04           | -6.19E-07          | 1.55E-05           | 2.98E-05           |
| 2     | 1.14E-03           | 9.34E-05           | 6.36E-04           | 1.27E-06           | 1.12E-05           | 6.10E-05           |
| 3     | 1.76E-03           | 1.56E-06           | 1.06E-03           | 9.36E-07           | 3.29E-05           | 2.81E-06           |
| 4     | 1.95E-03           | 2.27E-04           | 1.18E-03           | 3.69E-06           | 2.87E-05           | 1.53E-04           |
| 5     | 2.44E-03           | 3.46E-04           | 1.54E-03           | -9.67E-06          | 6.86E-05           | -2.99E-04          |
| 6     | 2.91E-03           | 4.64E-04           | 1.82E-03           | 1.03E-05           | 7.08E-05           | 3.40E-04           |

Figure 4.25: Boundary layer quantities calculated from RANS solution at $r/R= 0.6$ of propeller NSRDC4381 for $J=0.889$
Figure 4.26: Boundary layer quantities calculated from RANS solution at \( r/R = 0.95 \) of propeller NSRDC4381 for \( J=0.889 \)

| Point | \( |\delta_1^* / R| \) | \( |\delta_2^* / R| \) | \( |\theta_{11} / R| \) | \( |\theta_{22} / R| \) | \( |\theta_{12} / R| \) | \( |\theta_{21} / R| \) |
|-------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1     | 5.58E-04       | 2.01E-05       | 2.88E-04       | 4.44E-07       | 5.45E-06       | 2.41E-05       |
| 2     | 5.28E-04       | 4.62E-05       | 2.48E-04       | 1.50E-06       | 1.40E-05       | 2.63E-05       |
| 3     | 1.24E-03       | 3.38E-05       | 7.35E-04       | 1.45E-06       | 4.19E-05       | 2.80E-05       |
| 4     | 1.21E-03       | 1.71E-04       | 7.00E-04       | 6.88E-06       | 4.20E-05       | 1.11E-04       |
| 5     | 1.71E-03       | 9.42E-05       | 1.07E-03       | -7.69E-06      | 1.15E-04       | 7.57E-05       |
| 6     | 1.98E-03       | 3.78E-04       | 1.20E-03       | 2.22E-05       | 1.05E-04       | 2.48E-04       |

Figure 4.27: Boundary layer quantities calculated from RANS solution at \( r/R = 0.6 \) of propeller NSRDC4381 for \( J=0.5 \)

| Point | \( |\delta_1^* / R| \) | \( |\delta_2^* / R| \) | \( |\theta_{11} / R| \) | \( |\theta_{22} / R| \) | \( |\theta_{12} / R| \) | \( |\theta_{21} / R| \) |
|-------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1     | 9.38E-04       | 9.57E-05       | 5.12E-04       | 2.30E-06       | 3.00E-05       | 4.80E-05       |
| 2     | 1.83E-03       | 3.05E-04       | 9.98E-04       | 6.64E-06       | 3.28E-05       | 1.90E-04       |
| 3     | 1.58E-03       | 3.21E-04       | 9.49E-04       | 1.07E-05       | 5.01E-05       | 2.05E-04       |
| 4     | 2.73E-03       | 5.43E-04       | 1.65E-03       | 1.43E-05       | 7.78E-05       | 3.82E-04       |
| 5     | 2.35E-03       | 4.27E-04       | 1.47E-03       | 7.41E-06       | 7.19E-05       | 2.65E-04       |
| 6     | 3.90E-03       | 6.83E-04       | 2.47E-03       | 2.18E-05       | 2.32E-04       | 5.47E-04       |
4.6 Comparison with viscous-inviscid method

The streamwise boundary layer variables computed from RANS solution are compared with those from viscous-inviscid coupled method of Yu (2012) and results are shown in Figures 4.29 to 4.36. It can be seen that when the strip is near the tip for r/R=0.95, the boundary layer correlations are not good. This could be because calculating an accurate inviscid velocity distribution near the tip can be difficult due to geometric constraints. For other cases the correlations are reasonable but can be improved by adding the effects of viscous potentials of other strips.

![Table of Boundary Layer Quantities](image)

Figure 4.28: Boundary layer quantities calculated from RANS solution at r/R= 0.95 of propeller NSRDC4381 for J=0.5
Figure 4.29: Comparison of $\delta^r$ from RANS and a viscous-inviscid interactive solver for $r/R=0.6$ and $J=0.889$ on pressure side of propeller NSRDC4381.

Figure 4.30: Comparison of $\delta^s$ from RANS and a viscous-inviscid interactive solver for $r/R=0.6$ and $J=0.889$ on suction side of propeller NSRDC4381.
Figure 4.31: Comparison of $\delta^*$ from RANS and a viscous-inviscid interactive solver for $r/R=0.95$ and $J=0.889$ on pressure side of propeller NSRDC4381.

Figure 4.32: Comparison of $\delta^*$ from RANS and a viscous-inviscid interactive solver for $r/R=0.95$ and $J=0.889$ on suction side of propeller NSRDC4381.
Figure 4.33: Comparison of $\delta^*$ from RANS and a viscous-inviscid interactive solver for $r/R=0.6$ and $J=0.5$ on pressure side of propeller NSRDC4381.

Figure 4.34: Comparison of $\delta^*$ from RANS and a viscous-inviscid interactive solver for $r/R=0.6$ and $J=0.5$ on suction side of propeller NSRDC4381.
Figure 4.35: Comparison of $\delta^*$ from RANS and a viscous-inviscid interactive solver for $r/R=0.95$ and $J=0.5$ on pressure side of propeller NSRDC4381.

Figure 4.36: Comparison of $\delta^*$ from RANS and a viscous-inviscid interactive solver for $r/R=0.95$ and $J=0.5$ on suction side of propeller NSRDC4381.
Chapter 5

Ducted propellers with ducts having blunt trailing edge

5.1 Introduction

It is well known that inviscid boundary element method cannot solve flow around ducts with blunt trailing edge accurately since there is a large recirculation region of non-zero vorticity behind the duct. Since panel methods are inviscid they do not properly account for the recirculation region. To make the blunt duct section amenable to using the panel method, the blunt trailing edge needs to be modified to a sharp one using an extension scheme. The aim of the extension scheme should be to modify only the aft part of the duct without changing the geometry in the forward part. Pan (2009) proposed a scheme to extend hydrofoils of blunt trailing edge. Yu (2012) improved upon the scheme by using quadratic polynomials to model the extended shape. The method by Yu (2012) is improved upon and extended to three dimensional ducts with and without propeller.
5.2 Treatment of ducts with blunt trailing edge

5.2.1 Open duct case

In the extension method proposed by Yu (2012) for hydrofoils and propellers, the non-zero trailing edge needs to be well defined. For example, see Figure 5.1.

![Duct with well-defined blunt trailing edge](image)

Figure 5.1: Duct with well-defined blunt trailing edge. Taken from Yu (2012).

The starting point of the extension, which is modeled as a quadratic polynomial, is naturally known for such a section. However the duct used in this study does not have a well defined blunt trailing edge. Figure 5.2 shows the geometry of the duct used in this study.
For the geometry used in this study, the trailing edge near the inner part of the duct is not well defined. A scheme is proposed where the duct is cut by a plane at a particular axial location. The flap extension is modeled from the cut plane as shown in Figure 5.3.

The location of the cut plane can be changed as part of the improved extension scheme. In the new scheme, the horizontal position for the cut plane can be different for the outer and inner sides of the duct. This allows for more flexibility in selecting a desired extension according to the geometry of the duct section at hand. For the duct used in the current study, the outer side of the duct is almost horizontal so that separation is expected only on the inner side of the duct. Hence for this duct, the horizontal position of the cut plane for the outer side of the duct has been placed at the actual trailing edge location and

![Figure 5.2: Duct used in this study with blunt trailing edge along with proposed extension.](image)
Figure 5.3: Extension scheme for ducts with blunt trailing edge.

is not a variable. Only the horizontal position of the cut plane on the inner side of the duct, henceforth called as the ‘cut plane’ is varied to find a suitable extension. The vertical position of the modified trailing edge can be changed to get a different shape. The vertical position is changed until the pressures on both the sides of the duct at original trailing edge, that is at points A and B in Figure 5.3 are equal. The pressure equality condition is applied for the inviscid pressures in this thesis. The pressure equality condition can also be applied by using viscous pressures to obtain a more accurate extension shape.
Details of the quadratic extension scheme can be found in Yu (2012). Since the extension is not physical, the extended shape is required to satisfy equality of pressure at the actual location of the trailing edge. The extension should satisfy continuity of geometry and continuity of slope to get a smooth modified geometry as shown in Figure 5.4.

\[
\begin{align*}
y_C &= A_1 x_C^2 + B_1 x_C + C_1 \\
y_D &= A_2 x_C^2 + B_2 x_C + C_2 \\
y_E &= A_1 x_E^2 + B_1 x_E + C_1 \\
y_E &= A_2 x_E^2 + B_2 x_E + C_2 \\
slope @ C &= 2 A_1 x_C + B_1 \\
slope @ D &= 2 A_2 x_C + B_2
\end{align*}
\]

Figure 5.4: Quadratic expressions for modeling the extension.

The extension is basically an approximation of the recirculating region behind the duct. The cut plane location can be changed until a desirable shape is obtained.

5.2.2 Duct under the influence of propeller action

When the duct is under the influence of propeller, the pressures at different strips on the duct will be different. A scheme is proposed that applies the extension scheme described above, on the duct in an axisymmetric sense. For this, the pressure equality condition of the extension scheme is to be
satisfied for the circumferentially averaged pressure on the duct.

The geometry of the ducted propeller with the extended for $J=0.73$ is shown in Figure 5.5 and the paneling is shown in Figure 5.6.

Figure 5.5: Geometry of the ducted propeller with the hub and the duct wake.
Figure 5.6: Paneling on the duct, hub and the blade.
Also, since the extension is an approximation of region with high vorticity, the shape of the extension could be different for different rotational speeds of the propeller. Figures 5.7 to 5.10 shows the streamlines behind the duct for two different advance ratios. The streamlines are obtained by using an axisymmetric RANS-VLM coupled solver. In the axisymmetric RANS-VLM hybrid method, an inviscid Vortex Lattice method MPUF-3A is coupled with a viscous RANS solver FLUENT in an iterative manner. Since at the first iteration the effective wake is not known, it is assumed to be equal to the inflow velocity. The pressures obtained from the potential flow solver are time averaged and converted to body forces and added to the momentum equation in RANS as extra source terms. The viscous solver is used with these extra source terms to solve for the flow in an axisymmetric sense. The effective wake velocity at effective wake points is obtained by subtracting the propeller induced velocity from the potential flow solver from total velocity obtained from RANS solver. Once the new effective wake is obtained, the potential solver is run again to get the pressure forces and this process continues till convergence. More details about this method can be obtained in Kinnas et al. (2013).
Figure 5.7: Streamlines around duct for J=0.9 from RANS-VLM hybrid method of Kinnas et al. (2013).
Figure 5.8: Close view of streamlines near trailing edge of the duct for $J=0.9$ from RANS-VLM hybrid method of Kinnas et al. (2013).
Figure 5.9: Streamlines around duct for $J=0.3$ from RANS-VLM hybrid method of Kinnas et al. (2013).
Figure 5.10: Close view of streamlines near trailing edge of the duct for \( J=0.3 \) from RANS-VLM hybrid method of Kinnas et al. (2013).
The hydrodynamic performance prediction of the ducted propeller using the axisymmetric RANS-VLM method of Kinnas et al. (2013) is shown in the Figure 5.11.

Figure 5.11: Comparison of hydrodynamic performance prediction of the ducted propeller from experimental data and axisymmetric RANS-VLM method. Taken from Kinnas et al. (2013).

It can be seen from the figure, that the correlations between the results from axisymmetric RANS-VLM method and experimental open water test results are very good. Since the axisymmetric RANS-VLM method predicts the
performance accurately even for very low advance ratios, it was desired to do the same by coupling a potential flow method with an integral boundary layer solver. Hence it is proposed that a similar scheme can be used in the axisymmetric viscous-inviscid iterative method (Section 5.4), to predict the effects of viscosity on ducts of the ducted propeller.

5.3 Performance prediction for ducted propellers

In the current study three different cut plane locations were selected at 4 %, 6% and 8% of original chord length ‘c’ from the actual trailing edge location to get a modified duct shape so that panel method can be used. The intention is to couple the inviscid method with an integral boundary layer solver XFOIL. However, at this stage the separated zone behind the duct has been treated in an inviscid manner. The effect of friction forces on the duct is presently considered using an empirical friction coefficient $C_f = 0.0035$.

The extension scheme was used to obtain flap shape and the propeller performance characteristics were predicted using a perturbation potential based panel method. The extensions obtained using the different cut plane locations are shown in Figures 5.12 and 5.13.

It was found that when the advance ratio was changed, the modified shape of the duct obtained did not change significantly for a particular value of cut plane location. The current extension scheme is able to satisfy the conditions required for the extension for different values of cut location. In order to find the correct cut plane location for a particular advance ratio it
Figure 5.12: Modified geometry of the duct section obtained by applying the extension scheme at different cut-plane locations.
Figure 5.13: Close view of the extension obtained by applying the extension scheme at different cut-plane locations.
would be helpful to introduce an additional constraint on the extension in the future.

The results for three different flap shapes show that the extension do affect the performance characteristics of the ducted propeller. The duct pressure comparisons and blade circulation comparisons in uniform steady inflow are shown below in Figures 5.14 to 5.27.
Figure 5.14: Comparison of circumferentially averaged pressure on duct from panel method for different cut planes for defining duct extension, $J=0.3$.

Figure 5.15: Comparison of circulation on blade for different cut planes for defining duct extension, $J=0.3$. 

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Figure 5.16: Comparison of circumferentially averaged pressure on duct from panel method for different cut planes for defining duct extension, J=0.4.

Figure 5.17: Comparison of circulation on blade for different cut planes for defining duct extension, J=0.4.
Figure 5.18: Comparison of circumferentially averaged pressure on duct from panel method for different cut planes for defining duct extension, $J=0.5$.

Figure 5.19: Comparison of circulation on blade for different cut planes for defining duct extension, $J=0.5$. 
Figure 5.20: Comparison of circumferentially averaged pressure on duct from panel method for different cut planes for defining duct extension, $J=0.6$.

Figure 5.21: Comparison of circulation on blade for different cut planes for defining duct extension, $J=0.6$. 

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Figure 5.22: Comparison of circumferentially averaged pressure on duct from panel method for different cut planes for defining duct extension, J=0.73.

Figure 5.23: Comparison of circulation on blade for different cut planes for defining duct extension, J=0.73.
Figure 5.24: Comparison of circumferentially averaged pressure on duct from panel method for different cut planes for defining duct extension, $J=0.8$.

Figure 5.25: Comparison of circulation on blade for different cut planes for defining duct extension, $J=0.8$. 

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Figure 5.26: Comparison of circumferentially averaged pressure on duct from panel method for different cut planes for defining duct extension, J=0.9.

Figure 5.27: Comparison of circulation on blade for different cut planes for defining duct extension, J=0.9.
The predicted thrust and torque coefficients are compared with experimental data in Figures 5.28 to 5.30.

Figure 5.28: Hydrodynamic performance prediction for ducted propeller in uniform steady inflow with extension scheme having cut plane at 4 percent location.
Figure 5.29: Hydrodynamic performance prediction for ducted propeller in uniform steady inflow with extension scheme having cut plane at 6 percent location.
Figure 5.30: Hydrodynamic performance prediction for ducted propeller in uniform steady inflow with extension scheme having cut plane at 8 percent location.

It is noted that at higher advance ratios when the recirculation region is larger, the extension shape with larger length gives more reasonable predictions for the propeller thrust when compared to extension of smaller length. Similarly it can be seen that at lower advance ratios when the recirculation region is smaller, the extension shape with smaller length gives more reasonable predictions of propeller thrust when compared to extension of larger length. It
is also observed that there are more deviations in the predicted performance characteristics at lower values of advance ratio. This may be due to the fact that the empirical skin friction coefficient selected in this method may deviate significantly from its assumed value for highly rotational flows at low advance ratios.

5.4 Viscous-inviscid interactive method for ducted propeller

Following the observations made in Section 5.2.2, the viscous-inviscid interactive method is applied for the ducted propeller in an axisymmetric manner on the duct. The shape of the duct used is obtained from the extension scheme described in the previous section. The inviscid velocities, on a plane that passes through the axis and intersects the duct (meridional plane), at different strips are circumferentially averaged and this averaged velocity is used in the viscous-inviscid interactive method. When the rotational speed of the propeller is not very high, the streamlines along the duct are found to be almost axial. Figure 5.31 shows the streamlines on the duct and the wake for $J=0.73$. Therefore, the averaging was done on the velocity components along the meridional plane, since the loss of swirl component of the velocity during circumferential averaging was expected to be insignificant.

The comparison of viscous pressure on the duct as obtained by this method is shown with the results obtained form an axisymmetric hybrid RANS-VLM
Figure 5.31: Streamlines on the duct from panel method in ship fixed coordinate system for ducted propeller with $J=0.73$. 
Figure 5.32: Streamlines on the duct wake from panel method in ship fixed coordinate system for ducted propeller with $J=0.73$. 
solver by Kinnas et al. (2013) in Figure 5.33. It is to be noted that we are interested only in the results over the actual length of the duct. The results obtained on the extension beyond the location of the actual trailing edge are not of interest physically.

Figure 5.33: Comparison of circumferentially averaged pressure distribution on the duct from viscous-inviscid scheme for ducted propeller with RANS-VLM hybrid method of Kinnas et al. (2013), $J=0.73$. 

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However it is noted that for very low advance ratios, there will be more deviation from axial flow in the inner side of the duct due to propeller action. In such cases it may be required to use fully three dimensional boundary layer solver to correctly compute the effect of viscosity.
Chapter 6

Conclusions and Recommendations

6.1 Conclusions

In this thesis a three dimensional viscous-inviscid interactive method is presented wherein a two dimensional boundary layer solver is coupled with a three dimensional perturbation potential based solver. The viscous-inviscid coupling is applied in a stripwise fashion. It is assumed that the growth of the boundary layer is in only one direction. The formulation corrects the earlier formulation by Yu (2012) by taking into account the effects of other strips more accurately. It is shown that the neighboring strips not only affect the solution strip by means of blowing sources but also by the difference of viscous and inviscid potentials on other strips.

Contributions of the current work include:

• An improved formulation to predict the effects of viscosity on three dimensional geometries using two dimensional boundary layer solver in a stripwise manner. A simplified version of the formulation is presented for application to axisymmetric cases. The results predicted by the axisymmetric formulation for bare duct cases are in very good agreement with the results obtained from viscous RANS simulations. The viscous-
The inviscid method allows to correctly predict the frictional forces on the duct without using empirical friction coefficients. Also the effects of viscosity on the pressure distributions are correctly predicted because of the improved scheme. Besides, thorough grid convergence studies in chordwise and spanwise directions have established the method as robust and reliable.

- The method is applied to ducts of blunt trailing edges also after the trailing edge has been modified by an improved extension scheme. An improved extension scheme to obtain the flap shape has been presented. It carries forward the work by Pan (2009) and Yu (2012) and applies to open ducts and ducts under the influence of propeller. The effect of flap extension on the performance of ducted propellers is systematically investigated. It is found that the length of the flap extension predicting the performance more accurately follows the length of the recirculating region behind the duct. However depending on the cut plane location there may be multiple shapes satisfying the criteria imposed, so it is important to have a rational criterion to predict the location of the cut plane.

- The validity of the one directional boundary layer growth assumption is investigated for an open propeller by numerically evaluating the terms
in the fully three dimensional boundary layer equations and comparing
the streamwise terms with the spanwise terms. The study was done for
an open propeller at two different propeller rotational speeds.

6.2 Recommendations

• In this thesis a viscous-inviscid interactive method has been formulated
to solve for flow around three dimensional geometries in a stripwise man-
er. The method has been applied to case of three dimensional ducts
with sharp and blunt trailing edges to predict the effects of viscosity on
the duct. It is also important to obtain the effects of viscosity on the
propeller blade. To improve the performance prediction of ducted and
open propellers, the viscous-inviscid formulation developed here needs
to be applied to propeller blades in the future.

• For the case of ducted propellers, the viscous-inviscid interactive method
has been applied on the duct in an axisymmetric manner by circumfer-
entially averaging the inviscid velocity. However, the swirl component
of the velocity is neglected in this process. For flows at high blade ro-
tational speeds, there can be considerable swirl in the gap between the
duct and the blade tip. Such a flow would be highly three dimensional.
The streamlines also may not be axial along the duct section especially
after passing through the propeller zone at the inner side of the duct.
The validity of application of the viscous-inviscid interactive scheme in a
stripwise manner needs to be investigated for such cases. For a strongly three dimensional flow it would probably be best to use a three dimensional boundary layer solver for correct predictions. It would also be important to correlate the results of our method with those from a three dimensional RANS simulation for a ducted propeller for different loading conditions. Solution from RANS grid with adequate number of cells in the boundary layer will help to obtain the boundary layer characteristics in the streamwise and spanwise direction on the duct for different advance ratios as was done in the case of open propellers in this thesis. In this thesis the inflow was assumed to be steady, however for more practical applications, this method could be extended to unsteady and cavitating flows.

- The extension scheme by Yu (2012) has been improved and extended for application to three dimensional ducts with blunt trailing edge. In the current scheme, the cut location can be changed to get different extension shapes. It is recommended to add an additional constraint in the current extension scheme to select the correct cut plane location. The extension shape is an approximation of the area of separated flow behind the duct. The cut plane location can be selected at the point where the skin friction coefficient goes to zero, as that location will indicate the beginning of separation. The skin friction coefficient can be obtained by applying the viscous-inviscid interactive method on the duct with or
without the presence of propeller as presented in this thesis. Also while including the effects of viscosity on the ducts with blunt trailing edge, the separated zone approximation by extension scheme has been based on applying the pressure equality condition at the actual trailing edge location using inviscid pressures. For more accurate representation of the separated zone, it is recommended to use the viscous pressures for the pressure equality condition at the actual trailing edge location.
Appendix
Appendix 1

Evaluation of influence coefficients

In this appendix, the formulations of the influence coefficients for two dimensional and three dimensional panels are presented.

1.1 2D Influence coefficients

- Dipole
  \[ d_{ij} = \frac{1}{2\pi} \int \frac{\partial \ln r}{\partial n} dl \]  
  \[ (1.1) \]

- Source
  \[ s_{ij} = \frac{1}{2\pi} \int \ln r \, dl \]  
  \[ (1.2) \]

1.2 3D Influence coefficients

- Dipole
  \[ d_{ij} = -\frac{1}{4\pi} \int \frac{\partial (1/r)}{\partial n} dS \]  
  \[ (1.3) \]

- Source
  \[ s_{ij} = -\frac{1}{4\pi} \int \frac{1}{r} \, dS \]  
  \[ (1.4) \]

where, \( dl \) and \( dS \) are differential line and surface elements respectively.
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