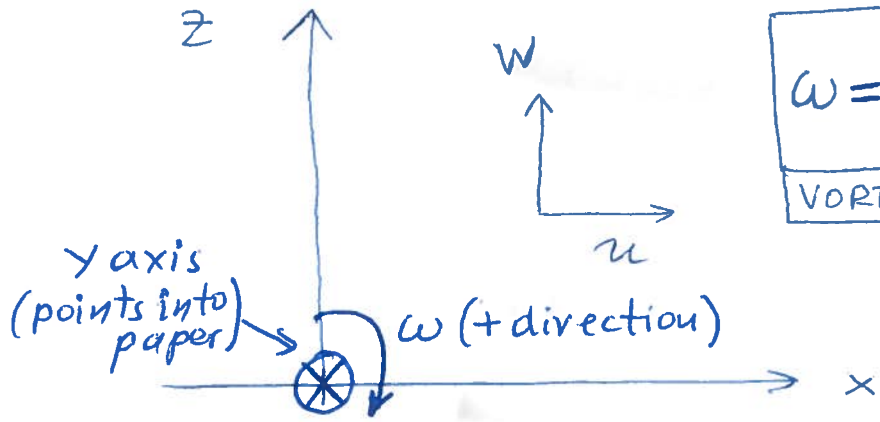
# Definition of vorticity (2-D flows):

(updated 9/4/2015)

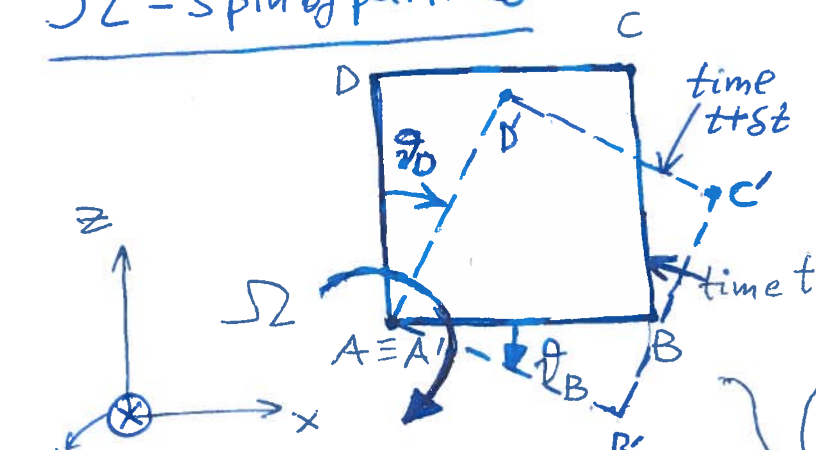



$$\boxed{\omega = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}}$$

VORTICITY $\omega$

Q: How is the vorticity, $\omega$, related to the angular velocity (spin) of a fluid particle?

$\Omega$ = spin of particle




The shown particle rotates with angular velocity $\Omega$ around point A (pure rotation).

Thus: $\vartheta_B = -\Omega . \delta t$

$\vartheta_D = +\Omega . \delta t$

(conventions for the signs for $\vartheta_B$ and $\vartheta_D$ are given in the webnotes section: "Deformation of fluid particle")

As shown in the webnotes (section "Irrotationality of flow")

$$\vartheta_B = \frac{\partial w}{\partial x}\delta t \quad \text{and} \quad \vartheta_D = \frac{\partial u}{\partial z}\delta t$$

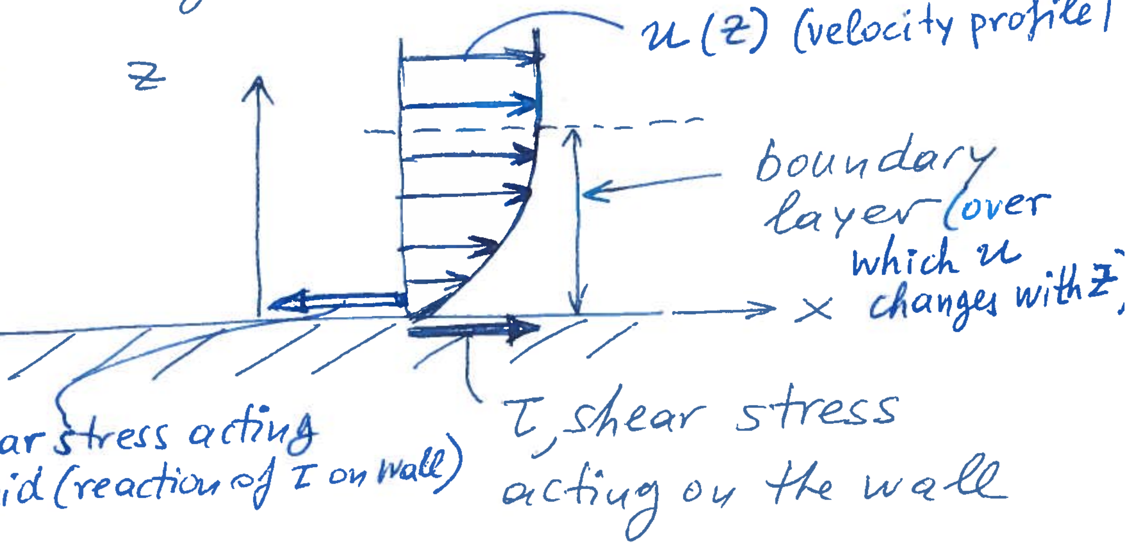
Thus vorticity $\omega = \dfrac{\vartheta_D}{\delta t} - \dfrac{\vartheta_B}{\delta t} = \dfrac{\Omega . \delta t}{\delta t} - \dfrac{-\Omega \delta t}{\delta t} = 2\Omega$

A: $\rightsquigarrow$ $\boxed{\omega = 2 . \Omega}$ $\Rightarrow$ VORTICITY IS DIRECTLY RELATED TO SPIN OF PARTICLE

vorticity — spin of particle

**Q:** Where in the flow is $\omega$ (or $\Omega$) significant?

Let's consider flow close to a solid boundary:



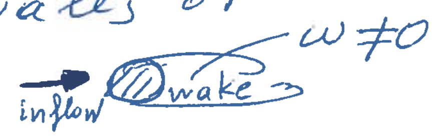

On the wall the vorticity is $\rightarrow \omega = \dfrac{\partial u}{\partial z}$ (why?)

However the shear stress $\tau$ is also given as: $\tau = \mu \dfrac{\partial u}{\partial z}$

$\uparrow$
(dynamic) viscosity

Thus $\omega = \dfrac{\tau}{\mu}$ or $\omega = \dfrac{\tau}{\mu}$

**A:** The vorticity in the flow will be significant ($\neq 0$) where $\tau$ is significant ($\tau \neq 0$). For example $\omega$ will be significant close to walls or in the wakes of bluff bodies



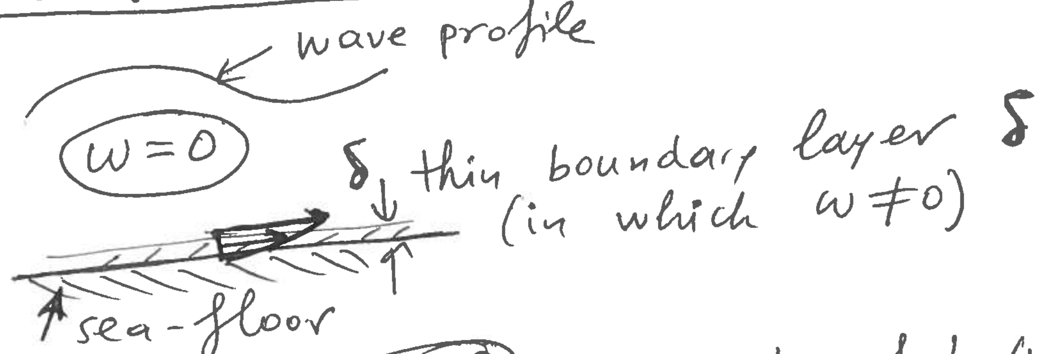
$Q_1$) When is $\underline{\omega\ (\text{vorticity}) = 2\,\Omega}$ valid?

spin of particle

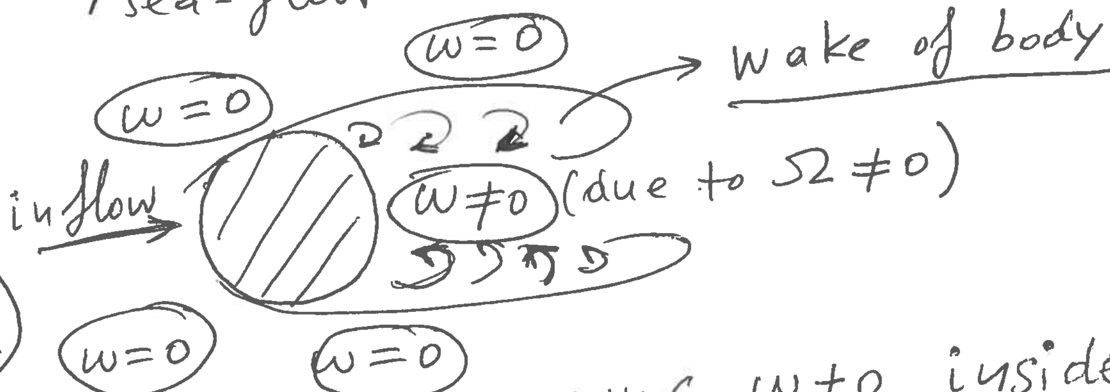
$A_1$) **ALWAYS!** In fact if we could measure the spin of a particle ($\Omega$) we would also know $\omega$ (vorticity)

Examples where $\omega = 0$ or $\omega \neq 0$:

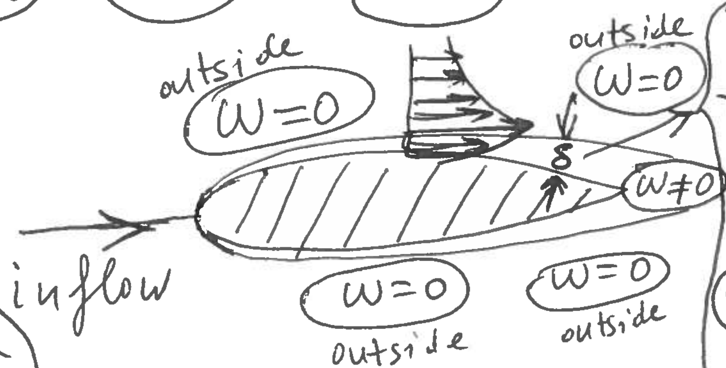
a) Wave flow




b) Flow around 2-D cylinder




c) Flow around streamlined object




$\omega \neq 0$ inside the thin boundary layer (in which velocity magnitude rapidly (with distance) reduces to zero, on the wall)