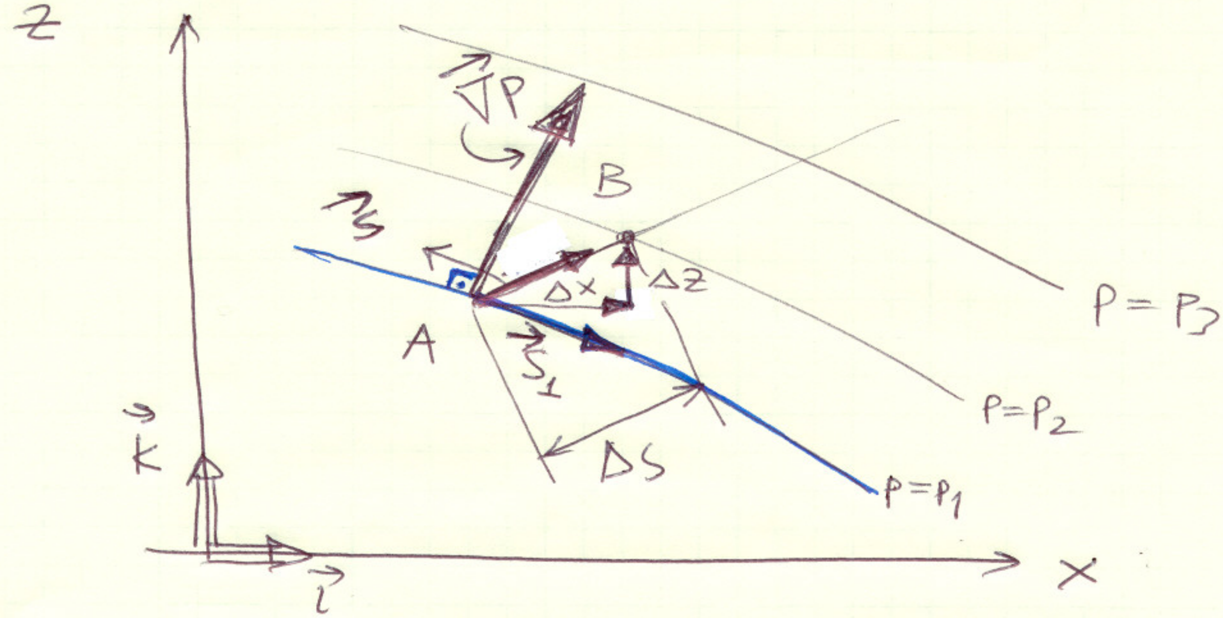
Physical meaning of gradient



We want to determine $\frac{\partial p}{\partial s} \approx \frac{\Delta p}{\Delta s}$

(change of p along direction AB)

$$= \frac{p_B - p_A}{\Delta s} = \frac{p(x_B, z_B) - p(x_A, z_A)}{\Delta s} \approx$$

$$\approx \frac{\frac{\partial p}{\partial x}\Delta x + \frac{\partial p}{\partial z}\Delta z}{\Delta s} = \underbrace{\left(\frac{\partial p}{\partial x}\vec i + \frac{\partial p}{\partial z}\vec k\right)}_{\substack{\vec\nabla p \\ \text{(definition of} \\ \text{gradient of } p)}} \cdot \underbrace{\left(\frac{\Delta x}{\Delta s}\vec i + \frac{\Delta z}{\Delta s}\vec k\right)}$$

$$\frac{\Delta x\,\vec i + \Delta z\,\vec k}{\Delta s} = \frac{\vec{AB}}{\Delta s} = \frac{\vec{AB}}{|\vec{AB}|} = \vec s \quad \text{(unit vector along AB)}$$

Finally $\boxed{\frac{\partial p}{\partial s} = \vec\nabla p \cdot \vec s}$

Apply above eq. for $\vec s_1$ (tangent to $p = p_1 =$ constant line) $\Rightarrow \frac{\partial p}{\partial s_1} = 0 = \vec\nabla p \cdot \vec s_1 = 0 \Rightarrow \boxed{\vec\nabla p \perp \vec s_1}$