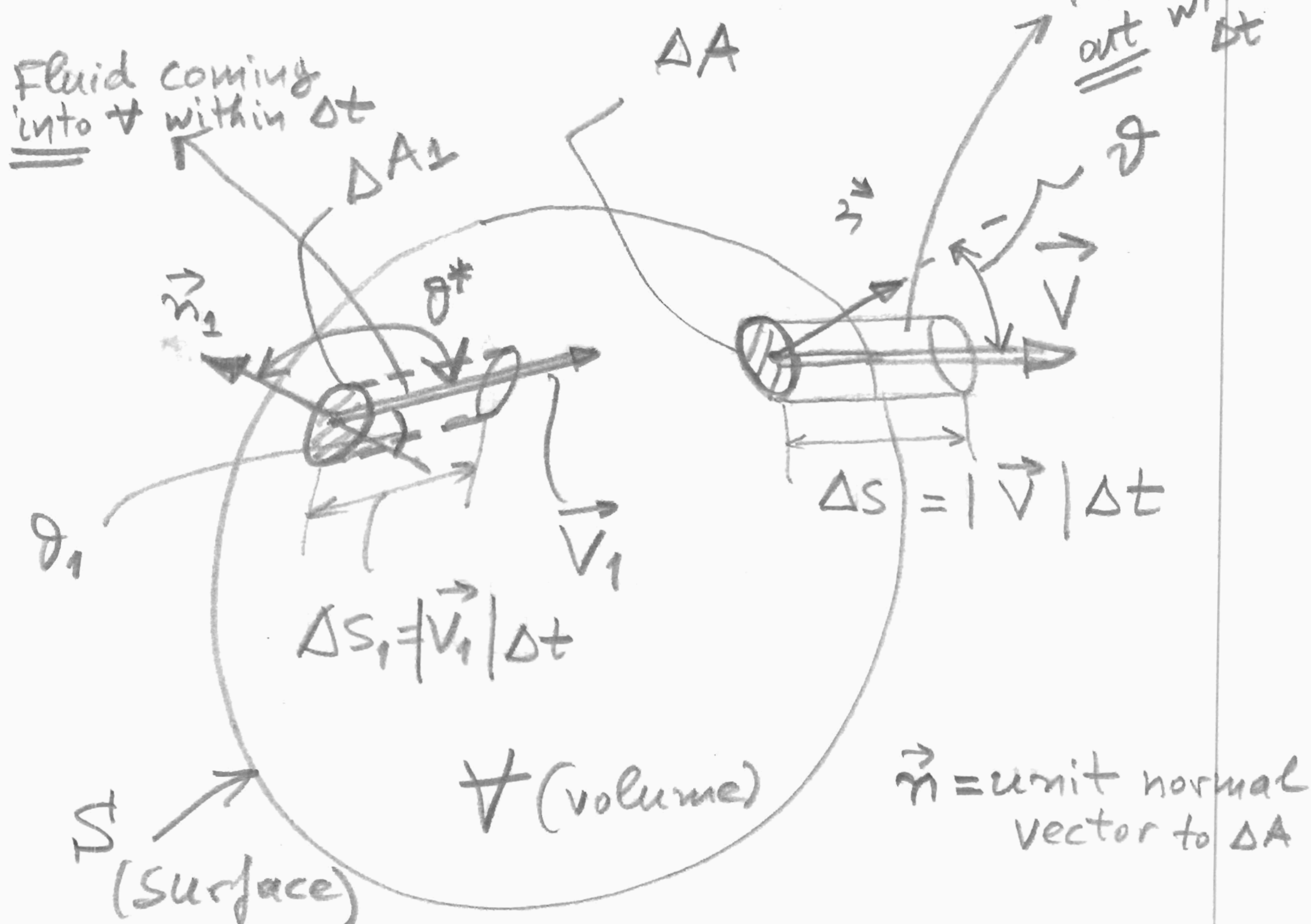
Definition of mass rate out of or into a volume $\forall$



Mass rate $\dot{m}_{\Delta A}$ out of $\forall$ through ΔA

Definition: $\dot{m}_{\Delta A} = \dfrac{\Delta m}{\Delta t} = \dfrac{\rho \, \Delta s \, \Delta A \cos\vartheta}{\Delta t} =$

$$= \rho \left(\frac{\Delta s}{\Delta t}\right)(\Delta A)\cos\vartheta = \rho \, |\vec{V}| \, (\Delta A) \cos\vartheta$$

But $\quad \vec{V}\cdot\vec{n} = |\vec{V}|\cdot\underbrace{|\vec{n}|}_{1 \text{ (unit vector)}}\cos\vartheta$

$$= |\vec{V}|\cos\vartheta$$

Thus $\quad \boxed{\dot{m}^{out}_{\Delta A} = \rho \, \vec{V}\cdot\vec{n} \, \Delta A} \quad (1)$

Mass rate $\dot{m}^{in}_{\Delta A_1}$ into the volume, $\forall$, through surface ΔA_1

$$\dot{m}^{in}_{\Delta A_1} = \frac{\Delta m}{\Delta t} = \frac{\rho \, \Delta S_1 \, \Delta A_1 \cos\theta_1}{\Delta t} =$$

$$= \rho \left|\vec{V}_1\right| \Delta A_1 \cos\theta_1$$

But $\vec{V}_1 \cdot \vec{n}_1 = \left|\vec{V}_1\right| \left|\vec{n}_1\right| \cos\theta^* =$

$$= \left|\vec{V}_1\right| \cdot 1 \cdot (-\cos\theta_1)$$

$$(\text{since: } \theta^* + \theta_1 = \pi)$$

So $\boxed{\dot{m}^{in}_{\Delta A_1} = -\rho \, \vec{V}_1 \cdot \vec{n}_1 \, \Delta A_1}$ (2)

Then above can also be written as

$$\dot{m}^{out}_{\Delta A_1} = -m^{in}_{\Delta A_1} = \rho \, \vec{V}_1 \cdot \vec{n}_1 \, \Delta A_1$$

So, we define $\boxed{\dot{m}^{out}_{\Delta A} = \rho \, \vec{V} \cdot \vec{n} \, \Delta A}$ (2')

The sign of the RHS of (2') will then tell us if the mass rate comes in or out of $\forall$