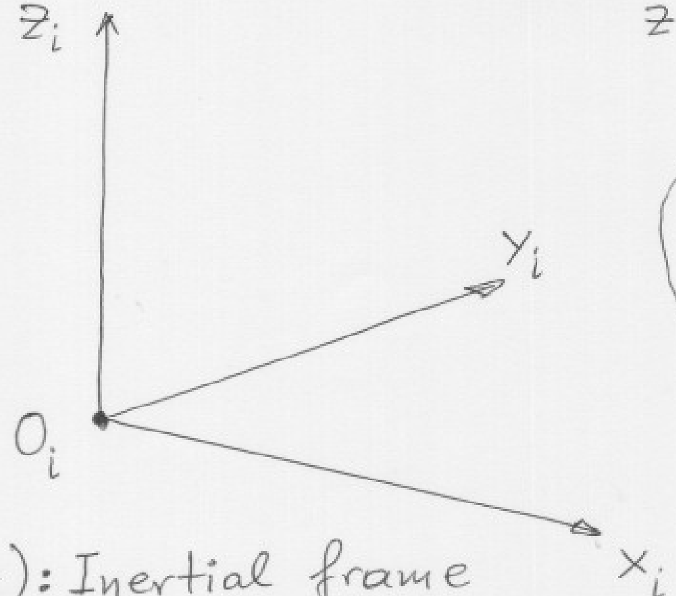
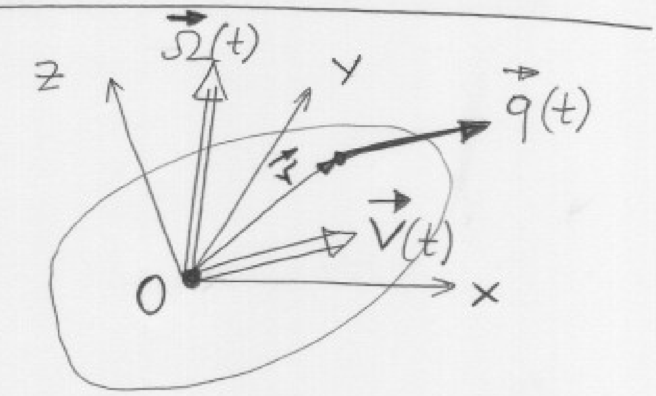
N-S equations w.r.t. moving frame

(for proof see "An Introduction to Fluid Dynamics" by G.K. Batchelor)





(Oxyz): Moving frame of reference

$(O_i x_i y_i z_i)$: Inertial frame of reference

$\vec{V}(t)$: tranlational velocity vector of (Oxyz)

$\vec{q}(t)$: flow field velocity vector w.r.t. moving frame

$\vec{\Omega}(t)$: angular velocity vector of (Oxyz)

$$\boxed{\frac{D\vec{q}}{Dt}} = \boxed{-\frac{\nabla p}{\rho} + \nu \nabla^2 \vec{q} + \vec{g}} \underbrace{- \frac{d\vec{V}}{dt}}_{\text{Tranlational acceleration of moving frame}} - \underbrace{2\vec{\Omega}\times\vec{q}}_{\text{Coriolis}} - \underbrace{\frac{d\vec{\Omega}}{dt}\times\vec{r}}_{\text{"No name"}} - \underbrace{\vec{\Omega}\times(\vec{\Omega}\times\vec{r})}_{\text{Centripetal}}$$

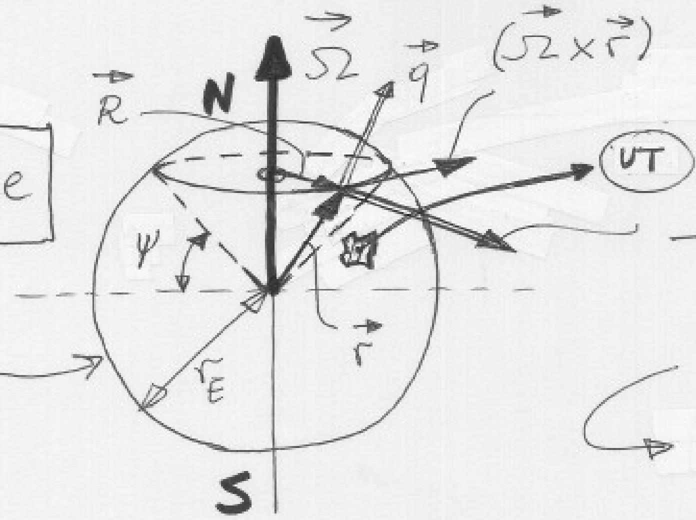
Same as if frame is not moving

Acceleration of fluid particle w.r.t. moving frame

Additional accelerations that have been put on the RHS with the (−) sign in front

Special application: Geophysical Flows

$$\vec{V} = 0 \quad \text{and} \quad \frac{d\vec{\Omega}}{dt} = 0$$



$$-\vec{\Omega} \times (\vec{\Omega} \times \vec{r}) = \vec{a}_{CF}$$

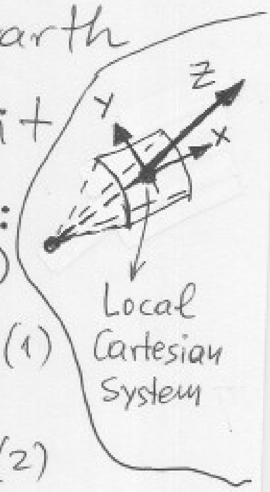
(centrifugal accel.)

$$\vec{a}_{CF} = \boxed{\Omega^2 \vec{R}}$$

N-S become:

$$\frac{D\vec{q}}{Dt} = -\frac{\nabla p}{\rho} + \nu \nabla^2 \vec{q} + \vec{g} - 2\vec{\Omega} \times \vec{q} \boxed{-\vec{\Omega} \times (\vec{\Omega} \times \vec{r})}$$

Assuming that $\vec{q}$ is tangent to the earth surface (i.e. vertical velocity $w = 0$) it can be shown that N-S reduce to:
(also ignoring the centrifugal acceleration)



$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - f v = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \nabla^2 u \quad (1)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + f u = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu \nabla^2 v \quad (2)$$

where f = Coriolis parameter or planetary vorticity $= 2\Omega \sin\psi$

Rossby number:

$$\boxed{Ro = V/(fL)}$$

For small $Ro \Rightarrow$ Coriolis effects important